THE PERCEPTIONS OF ELEMENTARY EDUCATORS REGARDING MATHEMATICAL DISCOURSE UTILIZATION IN THE CLASSROOM: A QUALITATIVE PHENOMENOLOGICAL STUDY

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A DISSERTATION

Presented to the Affiliated Faculty of
The College of Graduate and Professional Studies
at the University of New England

Submitted in Partial Fulfillment of Requirements
For the Degree of Doctor of Education

It was presented on
11/02/2023
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Date: November 2, 2023

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ABSTRACT

Given the lack of proper utilization of mathematical discourse in public elementary settings and the stagnation of mathematical proficiency, the purpose of this qualitative phenomenological study was to explore the perceptions of public elementary K-5 educators regarding the use of mathematical discourse as a pedagogical strategy. Because research indicates there are benefits to facilitating mathematical discourse, this study sought to understand the perceptions of public elementary K-5 teachers regarding the use of mathematical discourse as a pedagogical strategy in the classroom, their perceptions of different pedagogical strategies for facilitating mathematical discourse, and their perceptions of students engage in mathematical discourse while using disciplinary language. Through one-on-one semi-structured interviews with 10 participants, four themes emerged from the data: (a) teachers perceive mathematical discourse as positive, (b) classroom management impacts discourse facilitation, (c) the curriculum used and the professional development of teachers informs the strategies used to facilitate discourse, and (d) mathematical language plays a key role in student engagement in discourse. The findings indicate that districts and schools must provide further professional development on strategies to facilitate mathematical discourse for their teachers, select curricular programs that support teachers in facilitating discourse, and review their master schedules to ensure adequate time for mathematics instruction.

Keywords: mathematical discourse, elementary education, mathematics education

ACKNOWLEDGEMENTS

To my parents, Kathy and Rick Anderson, you have been so supportive as I embarked on this journey, even while going through your own challenges. Thank you for your unwavering support and for entertaining Charli while I worked on my classwork—to my dog, Charli, for being such a good girl!

To my lead advisor, Dr. Henke, thank you so much for your feedback and patience with me. Your feedback has been invaluable, and I would not have made it this far without your support and guidance.

To my secondary advisor, Dr. Bolden, thank you for your additional guidance and your enthusiasm over my topic. It is always exciting to meet someone else with a passion for mathematics education!

To Dr. Disque, thank you for reviewing my dissertation towards the end of my journey. You provided invaluable feedback.

Finally, to my classmates, thank you all so much for your feedback and our discussions throughout this entire process. You all helped my topic evolve and grow and helped make this achievable. To Kym, Rosalie, and Claire, my fellow advisees, thank you for your encouragement throughout this journey!

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CHAPTER 1: INTRODUCTION

Despite years of mathematics education reform in the United States (National Council of Teachers of Mathematics [NCTM], 2000; NCTM, 2014), the most recent international test scores compared to the scores of other nations continue to show significantly below-average performance in mathematics, with the nation falling 37th overall out of 79 participating countries (Organisation for Economic Co-operation and Development [OECD], 2019). According to the National Center for Educational Statistics (National Center for Educational Statistics [NCES], 2022a), student proficiency trended positively beginning in 1973 until 2012, when progress stalled. This sudden freeze in progress possibly was attributed to declining funding of public education or the adoption of new national standards at the time (Heiser, 2013). However, progress continued to stall (NCES, 2022b). While there are many factors that can impact how students attain mathematical proficiency (Collins, 2011), one factor that requires much more research, especially at the elementary level, is the use of mathematical discourse as a means of constructing understanding (NCTM, 2014). NCTM (2010) defines mathematical discourse as "the mathematical communication that occurs in a classroom" (p. 1) and notes that it can refer to communication between students or between student and teacher.

Existing research shows the benefits of mathematical discourse, including an increase in mathematical proficiency (Anderson, 2021; Bertolone-Smith & Gillette-Koyen, 2019; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021) and improvement in disciplinary language use (Croce & McCormick, 2019; Sigmon et al., 2022; Xu & Clarke, 2019). In fact, in its quest for mathematical reform and proficiency, NCTM (2014) stated that mathematical discourse was one of the key mathematics teaching practices that should be implemented in kindergarten through high school. Former

NCTM president Robert Q. Berry (2019) wrote, “It is imperative that we provide opportunities for each and every student to talk and to have their ideas heard during mathematics lessons” (para. 8), as doing so positions students as competent and builds mathematical understanding. According to NCTM (2014), all elementary educators should engage students in mathematical discourse as part of the action plan to improve mathematical proficiency.

Existing research also indicates there are pedagogical strategies used by teachers that foster classroom environments rich with mathematical discourse (Anderson, 2021; Bennett, 2014; Bertolone-Smith & Gillette-Koyen, 2019; Garcia et al., 2021; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Silva, 2021; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021; Wilson & Smith, 2022; Woods, 2021). These strategies include the use of scaffolding to support students, the use of cognitively demanding tasks, the setting of clear expectations for discussion and classroom culture, use of questioning, critiquing student work in thoughtful ways, and positioning students to be active participants and leaders of mathematical discussion, among others. If educators are to increase the amount of discourse occurring in the mathematics classroom, as NCTM (2014) suggests, these research-based strategies must be understood and utilized in more classrooms.

Additionally, as a mathematics educator in a public elementary setting, the research in this study allows the researcher to reflect on her own mathematical instruction and will encourage the maximization of best practices. This research may support other teachers who are looking to increase their opportunities to foster and provoke mathematical discourse in their classrooms among students. For those who are looking to increase opportunities to foster and provoke mathematical discourse among students in the classroom, this research also speaks to the importance of making it a daily focus of the mathematics educational experience. Thus, this study focuses on the phenomenon of mathematical discourse as it pertains to the elementary level, with the purpose of exploring the perceptions of public elementary educators about mathematical discourse within their own classrooms. Coded interviews were used to identify common themes, which provided practical implications for teachers seeking to improve their practices and researchers interested in continuing the study of mathematical discourse.

Definition of Terms

To understand the concepts within this study, several key terms are defined below to provide context.

Adaptive Reasoning. Students can make connections between mathematical concepts, and they can justify their thinking or adjust their thinking when appropriate (Collins, 2011).

Cognitively Demanding. In mathematics, higher-level or cognitively demanding tasks either require the making of connections between concepts or the exploration of concepts that require significant cognitive effort (NCTM, 2014).

Conceptual Understanding. Students understand when and why a mathematical idea is important (Collins, 2011).

Disciplinary Language. Language related to a subject area (i.e., mathematics), often encountered in school (Moore & Schleppegrell, 2020). In mathematics, language is used to understand problems and provide direction (Croce & McCormick, 2019).

Mathematical Communication. The expression of mathematical ideas through written, verbal, symbolic, visual, or abstract ways (NCTM, 2014).

Mathematical Discourse. Mathematical discourse refers to “the mathematical communication that occurs in a classroom” (NCTM, 2010, p. 1). That communication can be expressed in multiple ways and can occur from student-to-student or teacher-to-student (NCTM, 2010).

Mathematical Proficiency. For the Programme for International Student Assessment (PISA), proficiency is measured by six levels, with Level 1 being the lowest and Level 6 being the highest (OECD, 2019); in general, students are considered mathematically proficient when they can display the following strands of mathematical thinking: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Collins, 2011).

Procedural Fluency. Students can use procedures appropriately and flexibly, where appropriateness is determined by knowing when to use procedures and how to use them most efficiently (Collins, 2011).

Productive Disposition. Students believe themselves to be mathematicians and see mathematics as useful to 21st century learning (Collins, 2011).

Strategic Competence. Students can understand word problems, translate them into mathematical representations of their choice, and then solve those problems (Collins, 2011).

Statement of the Problem

The problem to be studied is the lack of proper utilization of mathematical discourse in public elementary settings (Fuson & Leinwand, 2023; Stiles, 2016), which is needed to increase mathematical proficiency on assessments like the Programme for International Student Assessment (PISA) and the National Assessment of Educational Progress (NAEP), both of which showed decreasing scores on the most recent administrations (NCES, 2022a; OECD, 2019). Rather, Fuson and Leinwand (2023) found many classrooms fail to use mathematical discourse in engaging and consistent ways, while Stiles (2016) found that most mathematics classrooms were still teacher-centered and therefore failed to support an environment conducive to discourse. Further, there is little research on how teachers perceive mathematical discourse, if they use it, and how they make instructional decisions during class. By examining teacher perceptions of mathematical discourse through qualitative phenomenological research, a better understanding of classroom communication in mathematics was obtained.

Mathematical discourse is defined by NCTM (2010) as “the mathematical communication that occurs in a classroom” (p. 1). Mathematical practice three of the Common Core State Standards (CCSS), as adapted from NCTM (2014), states that students need to be able to “construct viable arguments and critique the reasoning of others” (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010, p. 6) and is a skill developed at every grade level kindergarten to 12. At the elementary level, constructing arguments may occur through active means, where “each student builds his or her own mathematical knowledge from personal experiences, coupled with feedback from peers, teachers and other adults, and themselves” (NCTM, 2014, p. 9). These personal experiences fall under mathematical discourse or, in other words, mathematical communication.

NCTM (2014) discussed the decline of mathematical achievement in the United States in *Principles to Actions* and reported a plethora of issues plaguing the country. Included in this action plan was the claim that less than half of high school graduates were prepared academically for college-level mathematics, as well as the staggering statistic that “only 16 percent of U.S. high school seniors are proficient in mathematics and interested in a STEM career” (NCTM, 2014, p. 2). This concept of low achievement combined with non-existent growth summarized the problem in mathematics education in the United States for NCTM (2014).

Yet despite the adoption of the CCSS in 41 states (Common Core State Standards Initiative, 2022), and a push towards more communication to meet the rigor that the standards demand (NCTM, 2014), students in the United States continue to fail to perform on international standardized tests compared to students in other countries (OECD, 2019). In 2019, the OECD released the results of the 2018 PISA. The 2018 PISA assessed 15-year-olds globally in 79 countries (OECD, 2019). While reading was the focus of this most recent assessment, mathematical literacy was also assessed and presented in OECD's results. Results for math were scored on a leveled scale from 1 to 6, where 1 indicated a lower range of proficiency and 6 indicated a more complex range of proficiency. Students were considered proficient beginning with a score of 3 (OECD, 2019). Overall, the United States scored 37th out of the 79 countries and scored below the OECD's global average score. Approximately one fourth of students scored 1 or 2, indicating about one in four American students was not proficient in mathematics. Additionally, despite the work of NCTM and the CCSS in reforming mathematics, the overall trajectory of the United States flatlined, meaning there had not been a significant increase or decrease in scores between the first mathematics PISA administration in 2003 to the most recent assessment in 2018 (OECD, 2019). In fact, the overall score decreased from 2003 by 5 points.

It is difficult to make comparisons to countries with vastly different demographics and educational philosophies to those of the United States; however, national tests support a similar issue in mathematics. Most recently in 2022, the NCES (2022b) administered a special version of the National Assessment of Educational Progress (NAEP), an assessment used within the United States to determine proficiency levels of reading and mathematics. The test was administered to elementary-aged students following the COVID-19 pandemic and disrupted or remote learning. NCES (2022b) reported a statistically significant decline in mathematics scores compared to the 2020 NAEP scores, with the greatest declines in scores from students already performing below grade level. Scores at every percentile range decreased, even among the top percentage of students (NCES, 2022b). However, the mean score (234) remains slightly higher than the mean score (219) in 1973 when NAEP was first administered (NCES, 2022b).

Taking a more promising turn, research has indicated that mathematical discourse has benefits to the achievement and proficiency of students in mathematics (Anderson, 2021; Bertolone-Smith & Gillette-Koyen, 2019; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021). Martin et al. (2015) found that teacher questioning during instruction greatly impacted the level of discourse, and use of cognitively demanding tasks also allowed for greater discourse. Likewise, Ghousseini et al. (2021) explained that "children as early as kindergarten can consider alternative strategies and are capable of sophisticated mathematical thinking" (p. 364), and to promote this thinking, students must be able to communicate mathematically. They further argued that the classroom teacher is the one who must provide opportunities for this communication to take place (Ghousseini et al., 2021).

However, given the evidence that mathematical proficiency is not increasing (NCES, 2022a; OECD, 2019), especially in light of the NAEP (NCES, 2022b) results following the COVID-19 pandemic, information is needed about how teachers are promoting and how they perceive mathematical discourse in the classroom. Research exists that demonstrates the benefits of mathematical discourse in classrooms (Anderson, 2021; Bertolone-Smith & Gillette-Koyen, 2019; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021) and provides information on the pedagogical practices that maximize mathematical discourse (Anderson, 2021; Bennett, 2014; Bertolone-Smith & Gillette-Koyen, 2019; Garcia et al., 2021; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Silva, 2021; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021; Wilson & Smith, 2022;

Woods, 2021). Further research indicates that mathematical discourse also increases the use of disciplinary language by students, which allows them to access word problems (Croce & McCormick, 2019; Sigmon et al., 2022; Xu & Clarke, 2019). Given these benefits, it seems to follow that test scores should be progressing positively, yet they remain overall stagnant. There is also little research indicating whether or how educators are facilitating mathematical discourse in the classroom, nor how they perceive its use.

Purpose of the Study

The purpose of this qualitative phenomenological study was to explore the perceptions of public elementary K-5 educators regarding the use of mathematical discourse as a pedagogical strategy, where mathematical discourse is defined by NCTM (2010) as "the mathematical communication that occurs in a classroom" (p. 1). To achieve this purpose, the study also explored the educators' use of pedagogical strategies that facilitate mathematical discourse. Given the stagnant and decreasing scores seen on recent national and international assessments (NCES, 2022a; NCES, 2022b; OECD, 2019), one must consider the role of the teacher. As teachers are responsible for the delivery of and interaction students have with instruction (Buchheister et al., 2019), it is necessary to understand how mathematics educators approach mathematics instruction. One such approach is the use of mathematical discourse, or the mathematical communication that occurs in classrooms (NCTM, 2010). Mathematical discourse has been associated with increased mathematical proficiency (Anderson, 2021; Bertolone-Smith & Gillette-Koyen, 2019; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021) and as such, the research identified a path towards addressing the existing problem. An understanding of teacher perceptions regarding the use of mathematical discourse can also provide a way to reflect upon the current problem.

Research Questions and Design

Given the stated purpose of the study, there is one primary research question with two sub-questions that are explored using semi-structured interviews with teachers at public elementary schools. Participants were all classroom mathematics teachers in grades kindergarten through fifth with at least one year of experience teaching mathematics at their current grade level. Given that this was a phenomenological study, the research questions allowed the researcher to understand the perceptions of public-school educators relating to the phenomenon of mathematical discourse. The research aimed to answer the following questions through this research:

Main Research Question. What are the perceptions of public elementary K-5 teachers regarding the use of mathematical discourse as a pedagogical strategy in the classroom?

Sub-Question 1. How do public elementary teachers describe how they use different pedagogical strategies for facilitating mathematical discourse?

Sub-Question 2. How do teachers describe how students engage in mathematical discourse while using disciplinary language?

The methodology of the phenomenological study described above included semi-structured interviews with teachers at public elementary schools, selected from participating schools to represent a range of classrooms, K-5, in New England states that utilize the Common Core State Standards: Connecticut, Maine, New Hampshire, Rhode Island, and Vermont. Recruitment emails produced 10 participants who met the study criteria. They were interviewed using Zoom, and the interviews were transcribed verbatim using the Zoom auto-transcription feature. The transcripts were member-checked by the participants. The data analysis involved (1) preparing and organizing the data for analysis; (2) exploring the data through the process of coding; (3) using codes to develop descriptions and themes; (4) representing the findings through narratives and visuals; (5) interpreting the meaning of the results through personal reflection and use of literature; and (6) conducting strategies to validate the accuracy of the findings (Creswell & Guetterman, 2019).

Conceptual and Theoretical Framework

Ravitch and Riggan (2017) define the conceptual framework as "an argument about why the topic one wishes to study matters, and why the means proposed to study it are appropriate and rigorous" (p. 26). The conceptual framework is composed of three parts, including the researcher's personal interest leading to the study, the topical research, and the conceptual framework, where the personal interest is the researcher's role as an elementary mathematics educator, the topical research is existing literature themes, and the conceptual framework is rooted in NCTM's (2014) mathematics teaching practices and Vygotsky's (1978) sociocultural theory. There are three themes in the existing literature. First, existing literature shows that mathematical discourse leads to increased mathematical proficiency (Anderson, 2021; Bertolone-Smith & Gillette-Koyen, 2019; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021). Second, existing literature identifies specific strategies that promote mathematical discourse (Anderson, 2021; Bennett, 2014; Bertolone-Smith & Gillette-Koyen, 2019; Garcia et al., 2021; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Silva, 2021; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021; Wilson & Smith, 2022; Woods, 2021). Lastly, existing literature shows that mathematical discourse increases the level and use of mathematical disciplinary language (Croce & McCormick, 2019; Sigmon et al., 2022; Xu & Clarke, 2019).

An important component of the conceptual framework is the identification of the mathematics teaching practices, each of which describes best pedagogical practices (NCTM, 2014). These mathematics teaching practices, first identified in NCTM's *Principles to Actions* (2014), include (1) establish mathematics goals to focus learning; (2) implement tasks that promote reasoning and problem solving; (3) use and connect mathematical representations; (4) facilitate meaningful mathematical discourse; (5) pose purposeful questions; (6) build procedural fluency from conceptual understanding; (7) support productive struggle in learning mathematics; and (8) elicit and use evidence of student thinking. Further, the major theoretical framework for the present study is Vygotsky's (1978) sociocultural theory, which states that use of social interactions as a method for teaching and learning is critical (Allman, 2022). The most vital aspect of sociocultural theory is the zone of proximal development (ZPD), where the teacher provides scaffolding for students to attain a higher level of understanding, and the concept of the more knowledgeable other (MKO), from whom the child learns (Allman, 2022; Steele, 2001; Vygotsky, 1978). When a student is given a task that is within the ZPD, they are cognitively challenged with appropriate scaffolding, or support from the MKO, and this is where the student will learn (Allman, 2022). This is relevant to the discussion of mathematical discourse, especially based on the existing work of Ghousseini et al. (2021), who believes that teachers must provide scaffolding for students to become proficient in mathematical language. Allman (2022) also noted that language is a key component of this theory, further illustrating that the sociocultural theory is important for understanding how teachers encourage mathematical discourse, which has been shown to increase disciplinary language (Croce & McCormick, 2019; Sigmon et al., 2022; Xu & Clarke, 2019).

Assumptions, Limitations, and Scope

When conducting the study, several assumptions were made. First, not all schools teach mathematics using the same methods or curriculum. This study examined teacher perceptions of mathematical discourse as it was performed in the classroom at time of the interview, so the results varied based on the experiences of the participating educators. It was not expected that every teacher would encourage, view, or use mathematical discourse in the same way, given that this was not the case in studies like Ghousseini et al. (2021) and Bertolone-Smith and Gillette-Koyen (2019). However, there was also the possibility that participating teachers might employ similar strategies. Second, it was assumed that responses obtained from teachers accurately reflected their current professional practices and that participating educators responded honestly to all questions asked of them during the interview.

There were also some limitations to the study. Because this was a phenomenological study that explored teacher use of mathematical discourse deeply, the researcher sought only 10 participants. The participants were recruited from within New England, excluding Massachusetts as the Common Core standards used there are modified (Bauerlein et al., 2017). Creswell and Creswell (2018) identify four major limitations to utilizing interviews: (1) the information about the concept is indirect and influenced by the views of the person being interviewed; (2) the interview takes place in a separate place (for this study: Zoom) from the actual setting (for this study: the educator's classroom); (3) the presence of the interviewer could cause bias in the responses; and (4) the articulation and perception of the interviewees may be poor. It was possible to avoid the first limitation by completing a member check and confirming that the transcripts were accurately transcribed. The second limitation could not be avoided since interviews were conducted outside the actual setting. Additionally, the researcher's own biases might impact the interpretation of themes and results, especially as she is a mathematics educator. Asking prompting and clarifying questions helped with the fourth limitation, but reflection upon the interview transcripts revealed some missed opportunities.

The scope of the study involved interviewing teachers in Connecticut, Maine, New Hampshire, Rhode Island, and Vermont. The interviews took place between June 2023 and July 2023. The location of the study was virtual; however, participants were interviewed from within New England. Interviews were conducted via Zoom with audio recording on; all participants gave permission to be recorded. The sample of the study was elementary teachers in grades K-5 who had at least one year of experience teaching the current grade level. The interview was semi-structured to allow for a focus on mathematical discourse but also allowed for promptings for more information when needed.

The researcher is a classroom teacher who interviewed the other classroom teachers. The researcher is also on the board of the New Hampshire Teachers of Mathematics (NHTM). There was the potential to have conflict of interest or bias, especially if interviewing members of NHTM. This could have led to potential ethical or power-dynamic concerns that might have impacted the responses. If participants disclosed they were members of NHTM, the researcher would disclose her role and confirm that the participant would continue involvement in the study; no participants identified themselves as members of NHTM. Other power-dynamic concerns were limited because the researcher is also a classroom teacher in New Hampshire.

Rationale and Significance

The National Council of Teachers of Mathematics (NCTM) has been at the forefront of mathematics education reform over the past several decades, beginning with their *An Agenda for Action* in 1980. Most recently, in 2014, the organization released *Principles to Actions: Ensuring Mathematical Success for All* as the next chapter in providing guidance for what mathematics instruction should be. NCTM (2014) identified six guiding principles, which included (1) teaching and learning; (2) access and equity; (3) curriculum; (4) tools and technology; (5) assessment; and (6) professionalism. Embedded within these principles are the eight mathematical practices that are now used as part of the Common Core State Standards (CCSS), of which the concept of mathematical discourse is part (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). NCTM (2014) writes that students need to be provided with experiences in the classroom that allow them to "construct knowledge socially, through discourse, activity, and interaction related to meaningful problems" (p. 9). The same text outlines expectations for educators as described in the conceptual framework, wherein educators must facilitate discourse. NCTM (2014) explicitly expects discourse in classrooms, yet there is little research about its frequency and poor scores on national and international assessments (NCES, 2022a; OECD, 2019). Based on the information regarding the continued problems the United States faces in mathematics (NCES, 2022a; NCES, 2022b; OECD, 2019) and the lack of research regarding how teachers are using and making instructional decisions that impact the level of mathematical discourse, this study provided much needed information on the topic. The analysis was made using Vygotsky's (1978) sociocultural theory and the mathematics teaching practices from NCTM. With a better idea of how teachers facilitate mathematical discourse, teachers may be able to adapt strategies that others use to improve their practice.

Summary

Given the changing of standards and mathematical practices (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010; NCTM, 2010; NCTM, 2014) and the understanding of the sociocultural theory of learning (Vygotsky, 1978), the pedagogy of teachers has had to adapt, as well. Evidence suggests that increasing mathematical discourse in mathematics classrooms leads to an improvement in mathematical achievement in students (Anderson, 2021; Bertolone-Smith & Gillette-Koyen, 2019; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021). While this evidence is promising, it does not explore teacher perceptions on encouraging and facilitating mathematical discourse in their classrooms. Further, the benefits of mathematical discourse have not been seen since the release of updated guidelines from NCTM in 2014 based on national and international data (NCES, 2022a; OECD, 2019). By interviewing teachers based on the main research question and sub-questions and discussing how they use mathematical discourse at the elementary level through a sociocultural lens, implications can be made for the future of mathematical instruction. Therefore, the purpose of this qualitative phenomenological study was to explore the perceptions of public elementary K-5 educators regarding the use of mathematical discourse as a pedagogical strategy, where mathematical discourse is defined by NCTM (2010) as “the mathematical communication that occurs in a classroom” (p. 1).

CHAPTER 2: LITERATURE REVIEW

Given that the purpose of this qualitative phenomenological study was to explore the perceptions of public elementary K-5 educators regarding the use of mathematical discourse as a pedagogical strategy, where mathematical discourse is defined by NCTM (2010) as “the mathematical communication that occurs in a classroom” (p. 1), it was essential that a complete understanding of mathematical discourse, its role in improving the success and achievement of students mathematically, and its importance pedagogically be developed. This chapter examines the history of mathematical discourse, which developed as the result of efforts to reform mathematics from the National Council of Teachers of Mathematics (NCTM); the conceptual and theoretical frameworks for the study, based on the sociocultural theory from Vygotsky (1978); and the review of existing literature pertaining to mathematical discourse. The review of the literature includes the implications of mathematical discourse on proficiency levels (Anderson, 2021; Bertolone-Smith & Gillette-Koyen, 2019; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021); the pedagogical strategies that influence the use of and quality of discourse, including scaffolding and the selection of cognitively demanding tasks (Anderson, 2021; Bennett, 2014; Bertolone-Smith & Gillette-Koyen, 2019; Garcia et al., 2021; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Silva, 2021; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021; Wilson & Smith, 2022; Woods, 2021); and the language development impacts of discourse (Croce & McCormick, 2019; Sigmon et al., 2022; Xu & Clarke, 2019). Each of these themes appears in the existing literature regarding the topic of mathematical discourse but as a whole leave a major research gap in understanding how teachers facilitate mathematical discourse in their classrooms.

Given the evidence that mathematical discourse leads to increased proficiency (Anderson, 2021; Bertolone-Smith & Gillette-Koyen, 2019; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021), an understanding of what constitutes proficiency must be defined. The term *mathematical proficiency* can have many implications for classroom teachers and administrators, as well as for students themselves, since schools and students are compared using test scores that identify proficiency levels (NCES, 2022a; OECD, 2017). For the purposes of this study, students are considered mathematically proficient when they can display the following strands of mathematical thinking: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (Collins, 2011; Corrêa & Haslam, 2021; NCTM, 2021; NRC, 2001; Suh, 2007). Students demonstrate conceptual understanding when they can explain when and why a mathematical idea in important; they demonstrate procedural fluency when they can use procedures appropriately and flexibly, where appropriateness is determined by knowing when to use procedures and how to use them most efficiently given the context of a situation; they demonstrate strategic competence when they can solve word problems by translating the problem into mathematical representations; they demonstrate adaptive reasoning when they connect concepts and justify their thinking; and they demonstrate productive disposition when they can justify the use of mathematics in the real world (Collins, 2011). Each of these characteristics appears in the literature reviewed regarding mathematical proficiency. The results from existing studies indicate the benefits of encouraging mathematical discourse in an elementary setting but also suggest a need for further research into how teachers perceive mathematical discourse and how they use it (Anderson, 2021; Bertolone-Smith & Gillette-

Koyen, 2019; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021).

History of Mathematical Discourse

The need for further research into mathematical discourse stems from the mathematics reform over the past three decades (Martin et al., 2015). In 2010, the Common Core State Standards (CCSS) were released, which included the mathematical practices that must be demonstrated by students at every grade level from kindergarten through high school (NCTM, 2014). These standards are used today in 41 states (Common Core State Standards Initiative, 2022) and include the need for students to (1) make sense of problems and persevere in solving them; (2) reason abstractly and quantitatively; (3) construct viable arguments and critique the reasoning of others; (4) model with mathematics; (5) use appropriate tools strategically; (6) attend to precision; (7) look for and make use of structure; and (8) look for and express regularity in repeated reasoning (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). Mathematical practice 3, "construct viable arguments and critique the reasoning of others" (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010, p. 6), provides that students should be able to communicate mathematically with others, which at the elementary level may include use of manipulatives or pictorial models and less abstraction than may be seen at the secondary level (NCTM, 2014). Students still should be able to use models and tools to aid in argumentation, where they both present their mathematical arguments and critique those of others, as this allows them to demonstrate "deeper mathematical understanding" (NCTM, 2014, p. 24).

These standards for mathematical practices were adopted based on the work from the National Council of Teachers of Mathematics (NCTM), who in 2000 published *Principles and*

Standards for School Mathematics (hereinafter: *Principles and Standards*, 2000). This work led to the reform of mathematics, resulting in the CCSS math standards. *Principles and Standards* (2000) referred to a communication standard for preschool through 12th grade specifically, which stated that students should be able to communicate their understanding not only to teachers but also to their peers and other involved parties. Moreover, not only did they need to be able to communicate for themselves, but they also needed to be able to critique the reasoning of others, which became the basis for mathematical practice 3 (NCTM, 2014). *Principles and Standards* (2000) also demanded the use of disciplinary vocabulary. NCTM (2000) argued that each of these practices must be taught and "nurtured" (p. 62) by teachers, who would serve as guides toward stronger, richer mathematical communication.

NCTM followed up in 2014 with *Principles to Actions: Ensuring Mathematical Success for All* (hereinafter: *Principles to Actions*, 2014), which was a direct response to *Principles and Standards* (2000) and remains the most recent guiding document for mathematics from NCTM. NCTM (2014) began to refer to communication as mathematical discourse and argued that this was one of eight mathematical teaching practices (separate from the mathematical practices that students must demonstrate) meant to improve instruction. These mathematics teaching practices included: (1) establish mathematics goals to focus learning; (2) implement tasks that promote reasoning and problem solving; (3) use and connect mathematical representations; (4) facilitate meaningful mathematical discourse; (5) pose purposeful questions; (6) build procedural fluency from conceptual understanding; (7) support productive struggle in learning mathematics; and (8) elicit and use evidence of student thinking (NCTM, 2014, p. 10). The fourth mathematical teaching practice, *facilitate meaningful mathematical discourse*, states that discourse needs to occur between students and requires the analysis of others' work.

Building upon the work of *Principles and Standards* (2000), *Principles to Actions* (2014) outlined the range of features of discourse using descriptions of Level 0 to Level 3 discourse. At the highest level, Level 3 discourse includes a student-centered format, student-driven questioning (a step further than teachers asking probing questions), justification of answers, use of mathematical representations for other students' work, and support of other students (NCTM, 2014). This contrasts with Level 0 discourse, a much more teacher-centered format where the discussion is led entirely by the teacher with little input from students (NCTM, 2014). Each level requires more from the students, with the onus of learning and communication on them as opposed to on the teacher. Not only do teachers have to "facilitate meaningful mathematical discourse" (NCTM, 2014, p. 10), but they must also help students become more independent in managing their discourse.

Further, with the release of *Principles to Actions* (2014), more change struck mathematics with the hope of bridging the gap between *Principles and Standards* (2000) and the CCSS (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). Yet despite this push for reform and the encouragement of mathematical discourse as part of the requirements for mathematics instruction at every grade level, the mathematical achievement levels of the students in the United States stalled, with no significant progress in proficiency as compared to students in other countries since 2003 when such information began to be documented (OECD, 2019). Rather, according to the 2018 PISA results, one in every four students in the United States failed to demonstrate proficiency in mathematics (OECD, 2019), and in 2022, the first decline ever registered in mathematics was seen on the NAEP.

According to Martin et al. (2015), the failure of American studies to reach mathematical proficiency on national and international assessments indicates a renewed need for an understanding of mathematical discourse as it currently stands in the United States. Studies suggest that mathematical discourse increases mathematical proficiency (Anderson, 2021; Bertolone-Smith & Gillette-Koyen, 2019; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021), even while there are other factors that may lead to an increase in mathematical proficiency. Studies also identify the strategies that promote mathematical discourse, such as questioning (Anderson, 2021; Ghousseini et al., 2021; Martin et al., 2015; Smith et al., 2020; Sullivan, 2019), critiquing or pressing for reasoning and narration (Anderson, 2021; Bertolone-Smith & Gillette-Koyen, 2019; Ghousseini et al., 2021; Varhol et al., 2021; Woods, 2021; Xu & Clarke, 2019), and establishing expectations and positioning students as mathematical leaders (Bennett, 2014; Ghousseini et al., 2021; Smith, 2021; Sullivan, 2019; Wilson & Smith, 2022). Each of these strategies aligns with the expectations of both *Principles and Standards* (2000) and *Principles to Actions* (2014).

In addition to these pedagogical strategies, teachers must respond during lessons to provide scaffolding supports for students, which may include the use of anchor charts, sentence frames, or the integration of choice into lessons (Bertolone-Smith & Gillette-Koyen, 2019; Garcia et al., 2021; Ghousseini et al., 2021; Sigmon et al., 2022; Silva, 2021; Smith, 2021; Smith et al., 2020; Sullivan, 2019; Wilson & Smith, 2022; Woods, 2021). Further, studies indicate that mathematical discourse leads to greater vocabulary development (Garcia et al., 2021; Ghousseini et al., 2021; Sigmon et al., 2022; Silva, 2021; Smith, 2021; Wilson & Smith, 2022; Xu & Clarke, 2019). Lastly, the use of cognitively demanding tasks has been shown to lead to increased mathematical discourse in the classroom (Martin et al., 2015; Silva, 2021; Smith, 2021; Wilson

& Smith, 2022). Further research is needed, however, to understand how teachers use and encourage mathematical discourse in their classrooms.

The push toward increased mathematical discourse, in conjunction with the understanding that there is an existing problem with the lack of utilization of mathematical discourse in public elementary settings (Fuson & Leinwand, 2023; Stiles, 2016) based on the results of the PISA (OECD, 2019) and NAEP (NCES, 2022a), leads to the purpose of this qualitative phenomenological study: to explore the perceptions of public elementary K-5 educators regarding the use of mathematical discourse as a pedagogical strategy, where mathematical discourse is defined by NCTM (2010) as “the mathematical communication that occurs in a classroom” (p. 1). Through decades of reform, the need for discourse has remained (NCTM, 2000; NCTM, 2014). Because much of the discourse in classrooms is the result of teacher decision-making (NCTM, 2014), it is necessary to understand the perceptions of teachers about discourse. Ideally, educators should strive for a student-centered, discourse-based mathematics classroom (NCTM, 2014), but perceptions of elementary educators may vary based on pedagogical decision-making.

Conceptual and Theoretical Framework

In this section, the conceptual and theoretical framework for the present study will be developed. The purpose of the conceptual framework is to identify the relevance and significance of the study, as well as to set the context (Ravitch & Carl, 2021). In this case, the conceptual framework is composed of three parts, including the researcher’s personal interest leading to the study, the topical research, and the conceptual framework. The theoretical framework for the present study is Vygotsky’s (1978) sociocultural theory, partnered with NCTM’s (2014) mathematics teaching practices, which together provide an understanding of best practices of instruction.

Personal Interest

As an elementary mathematics educator, the researcher has a specific interest in the use of mathematical discourse, especially by other educators. In her district, specific professional development has been provided to teachers that encourages them to utilize mathematical discourse. As much as the researcher is purposeful in selecting pedagogical strategies to encourage discourse, she recognizes that not every classroom is the same. Other educators may make decisions before and during instruction that facilitate discourse in different ways. Further, educators may encourage their students to utilize more disciplinary language during these discussions in unique ways compared to their peers. By finding out what other educators do to facilitate mathematical discourse, the instructional practices of mathematics educators may improve. As such, the researcher sought to understand the effects of mathematical discourse on mathematical proficiency beyond her own classroom and district, the pedagogical strategies used to encourage discourse by other educators, and the use of mathematical language through discourse by using the sociocultural theory as a framework for understanding.

Topical Research

The curiosities mentioned appear as major themes in the literature and include mathematical proficiency, pedagogical strategies, and mathematical language. The existing research suggests that mathematical proficiency increases as a result of using mathematical discourse in classrooms (Anderson, 2021; Bertolone-Smith & Gillette-Koyen, 2019; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021), while further research points to specific strategies that promote mathematical discourse (Anderson, 2021; Bennett, 2014; Bertolone-Smith & Gillette-Koyen, 2019; Garcia et al., 2021; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Silva, 2021; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021; Wilson & Smith, 2022; Woods, 2021). Additionally, the use of discourse has been shown to increase the level of mathematical language used by students (Croce & McCormick, 2019; Sigmon et al., 2022; Xu & Clarke, 2019). To understand these themes, however, both NHTM's (2014) guiding mathematics teaching practices and the sociocultural theory (Vygotsky, 1978) must be understood.

As part of the conceptual framework, mathematics teaching practices are analyzed and discussed. These practices indicate a need to facilitate mathematical discourse in all education settings in kindergarten through twelfth grade (NCTM, 2014). These mathematics teaching practices claim the following as best pedagogical practices in mathematics: (1) establishing mathematics goals to focus learning; (2) implementing tasks that promote reasoning and problem solving; (3) using and connecting mathematical representations; (4) facilitating meaningful mathematical discourse; (5) posing purposeful questions; (6) building procedural fluency from conceptual understanding; (7) supporting productive struggle in learning mathematics; and (8) eliciting and using evidence of student thinking. When educators establish mathematics goals to focus learning, they support both their own decision making and student ownership of learning; the establishing of goals is also connected with higher performance from students compared to those where goals are not explicit (NCTM, 2014). By implementing tasks that promote reasoning and problem solving, teachers can increase student engagement and motivation (NCTM, 2014). The use of multiple mathematical representations allows for deeper understanding and greater problem-solving capabilities (NCTM, 2014). NCTM (2014) also claims that mathematical discourse is a "primary mechanism for developing conceptual understanding and meaningful learning of mathematics" (p. 30). The use of purposeful questions allows teachers to identify what students know and adapt accordingly (NCTM, 2014). NCTM (2014) also supports fluency, meaning that students can use multiple appropriate strategies to solve problems and can explain their solutions. Further, students should engage in productive struggle while supported by their teachers, turning them into problem solvers rather than students looking only for correct answers. Lastly, mathematics teachers use student understanding to adapt their instruction (NCTM, 2014). Ultimately, NCTM (2014) purports that engaging in these mathematics teacher practices is necessary to boost the mathematical understanding of students.

The fourth mathematics teaching practice explicitly notes that a best practice of mathematics pedagogy is facilitating discourse. However, even beyond the fourth mathematical teaching practice, facilitating meaningful mathematical discourse*,* each practice has relevance to the present study and the use of mathematical discourse in elementary schools. The mathematics teaching practices appear as part of the best practices identified in the existing literature and provide an understanding of how educators should teach mathematics. One of the pedagogical strategies identified in existing studies, for example, found that establishing individual expectations in the form of goals for students increased mathematical discourse in the classroom (Ghousseini et al., 2021). This is indicated by NCTM's (2014) first mathematics teaching practice. Others indicated that purposeful questioning strategies further promoted rich discussion that led to greater understandings of concepts (Anderson, 2021; Martin et al., 2015; Sullivan, 2019; Ghousseini et al., 2021; Smith et al., 2020), which is indicated by fifth mathematics teaching practice (NCTM, 2014). The selection of cognitively demanding tasks, or tasks that require a higher level of problem solving than other tasks, also impacted the level of discourse in classrooms (Martin et al., 2015; Smith et al., 2020; Wilson & Smith, 2022). This is indicated by the second and seventh mathematics teaching practices (NCTM, 2014). By understanding each of the mathematics teaching practices purported by NCTM (2014), the research can be driven by a foundation in best practices for mathematics.

Theoretical Framework

The sociocultural theory (Vygotsky, 1978) describes how students learn through social interaction (Allman, 2022) and provides insight into how teachers should teach to reflect their students' developmental needs (Steele, 2001). In exploring the teaching of mathematics, especially when using mathematical discourse, educators must be aware of the social nature of learning (Steele, 2001). Allman (2022) presents sociocultural theory as "the role social interaction and culture play in the development of higher-order thinking skills" (para. 1). The theory comes from Russian psychologist Lev Vygotsky (1978), who outlined three parts of this development theory: the importance of social interaction, the use of language in learning, and the zone of proximal development. Vygotsky's (1978) sociocultural theory accounts for variations in child development that other theories, such as the theory of cognitive development (Piaget, 1936), do not. That is, development may not happen at the same age for the same children in the same location, but rather the child is influenced by the environment and the people within that environment (Allman, 2022).

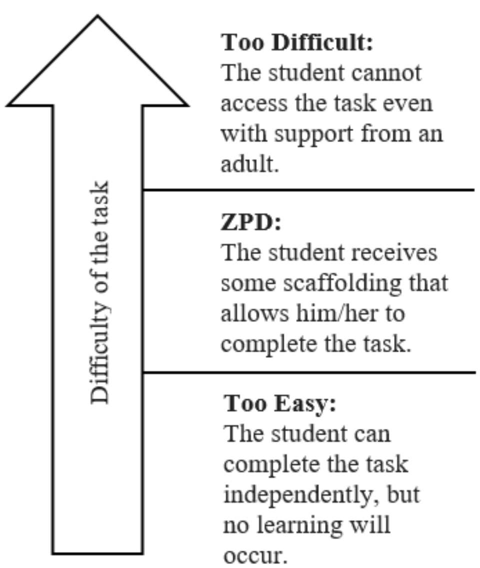
Sociocultural theory posits that social interaction is a key aspect of a child's cognitive development (Allman, 2022). According to Vygotsky (1978), the child will learn based on social interactions with a more knowledgeable other (MKO), such as a teacher, parent, or peer. By working collaboratively with the MKO within the appropriate instructional level, the student will internalize the strategies promoted by the MKO. The child is guided through activities with the MKO and over time can use the strategies from those interactions in new situations (Allman, 2022; Steele, 2001). Mathematically, this may mean the MKO (often the teacher or classmates) works with the student or models the mathematics, and over time through repeated interactions, the student internalizes the mathematics and can use it in a novel situation (Deogratias, 2022). Steele (2001) explains that mathematical understanding can only improve when students connect previous understanding with "new mathematical language" (para. 6) through discourse. In other words, when students work with their teachers and use mathematical language through repeated interactions, the student will be able to develop their mathematical understanding further (Deogratias, 2022; Steele, 2001).

Further, language plays a significant role in a child's cognitive development according to sociocultural theory (Allman, 2022; Steele, 2001). The sociocultural theory explains that language is used to assign meaning and, eventually, will be internalized as private speech, wherein the child can socialize to come to conclusions (Allman, 2022). According to Allman (2022), Vygotsky believed that "human action on both the social and individual planes is mediated by tools and signs, or semiotics, such as language, systems of counting, conventional signs, works of art, etc." (para. 12). That is, how humans respond to situations is based on the knowledge derived from these forms of cultural language. As children learn to internalize their language as private speech, they tackle more complex tasks through reasoning; thus, language and the ability to use language is important in the process of learning as children begin to use private speech (Allman, 2022). Children need the opportunity to discuss their processes and familiarize themselves with the semiotics of the topic so they may turn to private speech eventually and reason through more complex problems (Allman, 2022). Steele (2001) supports this in mathematics, explaining that children combine the information they learn and use it to create their own meaning as generalizations, or the private speech Vygotsky (1978) describes. In the study by Steele (2001), the researcher found that the teachers in the study used phases of learning to maximize that generalization process, with highly social collaboration and critique repeatedly used, which led to student generalization of concepts all based on the sociocultural theory.

Sociocultural theory also defines Vygotsky's (1978) zone of proximal development (ZPD). The ZPD is a concept that states there is a zone between two points where independent learning occurs and where the student cannot do the assigned task even with support. The area between the two points is the Zone of Proximal Development, wherein some support is needed from the MKO for the child to access the task, but the child must push themselves beyond what they can do independently, which is how learning happens (Vygotsky, 1978). This is seen below in Figure 1. The ZPD is different for every child, and as such, the scaffolding needed to support them will be different (Allman, 2022).

Figure 1

The Zone of Proximal Development



Note. This figure was created based on Vygotsky's description of the ZPD (1978).

Based on this explanation of the ZPD, the implication for mathematics is that different students will need different scaffolds in place to support them, given that each child has a different skill set that develops at different paces (Vygotsky, 1978). These supports may exist in the form of tools or additional language supports (Steele, 2001). In fact, Steele (2001) examines the ZPD in her study and reports that, while the teacher does not specifically refer to her scaffolds as the ZPD to students, the use of language scaffolds and checks for understanding allowed students to construct their understanding more fully. This aspect of sociocultural theory, the ZPD and use of scaffolds, is highly valuable to mathematical discourse because scaffolds are needed to support learners of various levels (Bertolone-Smith & Gillette-Koyen, 2019; Garcia et al., 2021; Sigmon et al., Silva, 2021; 2022; Smith et al., 2020; Sullivan, 2019; Wilson & Smith, 2022; Woods, 2021).

The major facets of sociocultural theory, that learning is a social process and that students learn best when provided with scaffolds within their ZPD, form the foundational understanding of the present study. That is, to understand how mathematical discourse is used by teachers, one must understand the role of social interaction in learning (Allman, 2022; Steele, 2001; Vygotsky, 1978). Further, sociocultural theory also makes it clear why the pedagogical strategies used by teachers are crucial for mathematical discourse because these strategies allow for greater access within the ZPD for students (Steele, 2001). Disciplinary language development, as well, relates to sociocultural theory, as this language is a key aspect to learning mathematics through discourse and can lead to private speech (Allman, 2022). Steele (2001) sums this up by saying, “As they learn to speak the mathematical language, they transform their thinking of the mathematical concepts” (para. 4). In other words, to become more proficient in mathematics, students need to grasp the mathematical disciplinary language, which comes through social learning and discourse.

While sociocultural theory explains how students learn, it also implies how educators should instruct (Das, 2020). Das (2020) writes, “To comprehend the higher mental functions of a person as a developmental process, the teacher or researcher needs to encourage the process” (p. 106). That is, the educator needs to take a sociocultural approach of social interaction as a method of learning for students to be able to learn and develop a stronger understanding of mathematical concepts. While Das (2020) also admits that the educator may not be approaching instruction while thinking specifically of sociocultural theory, the approach is embedded naturally in rich mathematics instruction. When combined with NCTM’s (2014) mathematics teaching practices, it is evident why mathematical discourse is so vital.

Mathematical Proficiency

The review of the literature explores current studies pertaining to mathematical discourse and critiques the gaps in the research. First, an understanding of the role of mathematical discourse in increasing proficiency in mathematics is explored. Pedagogical strategies, including scaffolding and the selection of cognitively demanding tasks, that can be used to promote mathematical discourse will be explored. Lastly, there will be an exploration of how discourse can improve mathematical language development. These themes provide a deeper understanding of previous research and provide context for the purpose of this study.

The importance of mathematical proficiency at the elementary level was first claimed by the National Research Council (NRC; 2001) in the critical text, *Adding It Up*, in which the authors state, “For both reading and mathematics, children’s performance at the end of elementary school is an important predictor of their ultimate educational success” (p. 18).

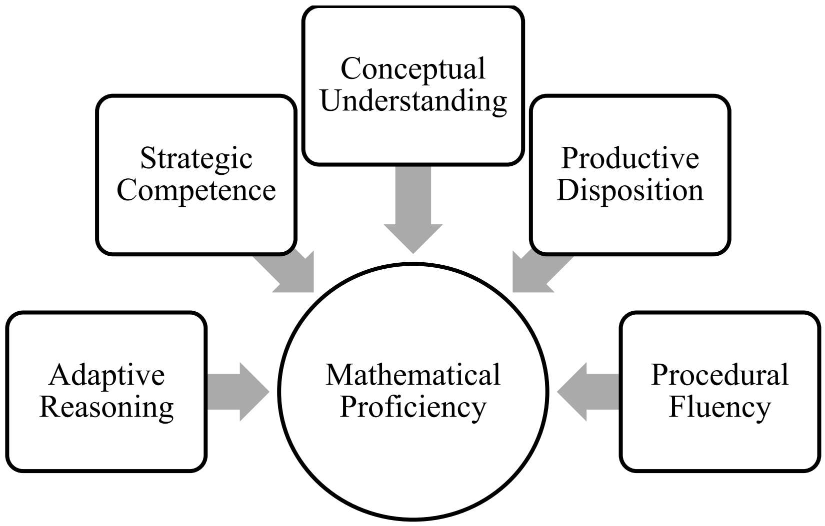
Researchers and NCTM continue to cite *Adding It Up* (2001) today as a critical factor in how we assess proficiency (Groves, 2012). In fact, the model of proficiency identified by the NRC (2001) was so influential that other countries such as Australia and South Africa adapted the model, as well (Groves, 2012). Mathematical proficiency stems from cognitive changes upon multiple strands that strengthen students as they progress educationally (Collins, 2011; Corrêa & Haslam, 2021; Groves, 2012; NCTM, 2021; NRC, 2001; Suh, 2007). These five strands must be interwoven pieces of mathematical proficiency, and students cannot be considered mathematically proficient without demonstrating mastery in all five areas (Collins, 2011; Corrêa & Haslam, 2021; Groves, 2012; NCTM, 2021; NRC, 2001; Suh, 2007).

The Five Strands of Mathematical Proficiency

Mathematical proficiency can be measured in a variety of ways (Groves, 2012), but it is acknowledged by mathematics educators and researchers alike that there are five strands of mathematical thinking that indicate a student has reached proficiency (Collins, 2011; Corrêa & Haslam, 2021; Groves, 2012; NCTM, 2021; NRC, 2001; Suh, 2007). The five strands that make up mathematical proficiency include adaptive reasoning, strategic competence, conceptual understanding, productive disposition, and procedural fluency (Figure 2). The five strands must work together and, as a whole, contribute to a student's overall proficiency.

Figure 2

The Five Strands of Mathematical Proficiency



Note. Based on the five strands of mathematical proficiency from *Adding It Up: Helping Children Learn Mathematics* (NRC, 2001).

First, a student is mathematically proficient when they demonstrate conceptual understanding in that the student can understand when and why a mathematical idea is important (Collins, 2011; Corrêa & Haslam, 2021; Groves, 2012; NCTM, 2021; NRC, 2001; Suh, 2007). To demonstrate conceptual understanding, the student needs to be able to understand the mathematical ideas being used, or represent the concepts through multiple representations, which also increases retention (Groves, 2012). This conceptual understanding forms the foundation needed to master later skills in mathematics. A study by Russell et al. (2020) found that using cognitively demanding tasks increases students' conceptual understanding. Further, they make the claim that building conceptual understanding "is critical in the current policy environment that has set rigorous college- and career-readiness standards, as the learning goal for all students" (Russell et al., 2020, p. 459).

Second, a student is mathematically proficient when they demonstrate procedural fluency, or the ability to use procedures appropriately and flexibly (Collins, 2011); knowing when and how to use procedures is a critical skill. It is rooted deeply in solid number sense (Groves, 2012). Procedural fluency is needed to gain conceptual understanding of some skills, indicating the need for the mastery of these strands in concert with each other rather than as separate strands (Groves, 2012; Suh, 2007). Procedural fluency is often mistakenly maximized in mathematics classes while the other strands are minimized (Groves, 2012; Suh, 2007). This is especially true because of the historical reality and current perception of assessments only measuring students' ability to perform procedures (Corrêa & Haslam, 2021). However, procedural fluency should only be one piece of assessment data and assessed in conjunction with the other strands. In fact, Graven and Stott (2012) argue that procedural fluency is a spectrum, and it is more important to view students as on the spectrum than as fluent or non-fluent. In their study, Graven and Stott (2012) found that as procedural fluency increased on the spectrum, students also developed stronger conceptual fluency, demonstrating the interconnectedness of the strands and their importance of developing them together. NCTM (2021) also promotes the interconnectedness of the strands by stating that content (the procedure) is just as important as the content.

Third, a student is mathematically proficient when they demonstrate strategic competence, or an understanding of word problems that can be translated into mathematical representations and solved (Collins, 2011). This is also known as mathematical problem solving (Groves, 2012). Groves (2012) writes, "Problems need to be sufficiently challenging to interest students, but not so difficult that they get frustrated" (p. 133). This is what Vygotsky (1978) purports with the Zone of Proximal Development (ZPD). Mathematical learning will take place within the ZPD when problems are sufficiently challenging. The teacher, as the more knowledgeable other (MKO), provides support when needed in the form of scaffolds (Vygotsky, 1978) but provides only enough support that the answer is not given away and the student can still solve it without frustration (Groves, 2012). Rather than simply pulling the numbers out of a cognitively demanding task, or a task that requires high effort and can have multiple solution pathways (Collins, 2011), students who have strategic competence develop strategies that assist them in understanding the task and solving challenging problems (Groves, 2012). Suh and Seshaiyer (2016) also state that improving strategic competence increases conceptual understanding and students' abilities to participate in discourse.

Fourth, a student is mathematically proficient when they demonstrate adaptive reasoning, or the ability to make connections between mathematical concepts and justify their thinking (Collins, 2011). Groves (2012) describes it as the ability to explain reasoning and justify thinking. Muin et al. (2018) argue that students need to be provided with non-routine problems in which they must adjust their thinking to the demands of the problem. They suggest that students use creative problem solving, wherein students practice "identifying challenges, creating ideas, and implementing innovative solutions" (p. 2). In their study, Muin et al. (2018) found that students who displayed more creative problem solving demonstrated strong adaptive reasoning skills, which also allowed them to outperform the group of students who exhibited more conventional problem-solving methods.

Lastly, a student is mathematically proficient when they demonstrate a productive disposition, or the belief that he or she is a mathematician and can use mathematics in 21st century learning to be successful (Collins, 2011). Students need to see the usefulness of mathematics in their everyday lives (Groves, 2012). Groves (2012) also states that this strand is often ignored on larger scales, despite the emphasis placed on it by educators. Mathematical achievement also has been connected to productive dispositions where students with greater productive dispositions have higher achievement (Suh, 2007). However, Corrêa and Haslam (2021) note that productive disposition is the most difficult of the strands to observe. They claim that the teaching practices utilized by the educator play a key role in developing productive dispositions in students (Corrêa & Haslam, 2021).

With these five strands interwoven, a student is more likely to be mathematically successful (Collins, 2011; NRC, 2001; Suh, 2007). Suh (2007) indicated in her research that by focusing purposefully on the five strands, her students became stronger problem solvers and saw mathematical connections to the real world. Of course, educators need to be able to determine whether students are mathematically proficient (Corrêa & Haslam, 2021). Therefore, how mathematical proficiency is measured also provides implications for how mathematics should be taught, including use of discourse (Collins, 2011).

Measurement of Proficiency

The assessment of mathematical proficiency needs to be rooted in the five strands, as well, rather than focusing on procedures as it has been in the past (Corrêa & Haslam, 2021). Open-response problem solving tasks are ideal for measuring mathematical proficiency over standardized tests (Corrêa & Haslam, 2021). While the Programme for International Student Assessment (PISA) is considered a standardized test, OECD (2017) claims that the assessment measures beyond procedural fluency and looks at "students' capacity to formulate, use and interpret mathematics in a variety of contexts" in real world situations. Similarly, the National Assessment of Educational Progress (NAEP) claims to measure proficiency for problem solving rather than solely procedural fluency and use a range of complexity in the tasks (NCES, 2020b).

As suggested by Corrêa and Haslam (2021), assessments are adapting to the understanding of mathematical proficiency.

However, Corrêa and Haslam (2021) suggest using classroom-level assessments such as observations and purposeful tasks as the main assessments used to determine mathematical proficiency. Collins (2011) claims that use of formative assessment to measure mathematical proficiency "positively affects students' achievement" (p. 1). In fact, discourse and collaborative work are key characteristics of classrooms that effectively use formative assessment (Collins, 2011). Discourse itself can be used as formative assessment, and when formative assessment is used effectively by teachers, even more discourse can occur (Collins, 2011).

The Impacts of Discourse on Proficiency

NCTM's *Principles and Standards* (2000) and *Principles to Actions* (2014) both highlight the need to encourage mathematical discourse at all levels. This push towards increased discourse is not only necessary according to NCTM, but it is also supported by research that states that through mathematical discourse, deeper understanding of mathematical topics can be achieved (Anderson, 2021; Bertolone-Smith & Gillette-Koyen, 2019; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021). In fact, Bertolone-Smith and Gillette-Koyen (2019) make the bold claim that mathematical discourse is so important that it impacts multiple levels of development, including social, emotional, and mathematical development. Because of this, there must be existing foundations in place to support this development, as per the constructivist perspective, which states that for learning to occur, students need existing foundational knowledge upon which to build (Narayan et al., 2012).

At a higher level, students learn to correct misunderstandings, which also indicates higher cognitive ability (Bada, 2015; Narayan et al., 2012). With mathematical discourse, studies found that students are better able to self-correct when pressed for reasoning, a form of mathematical discourse in which teachers and peers use questioning scaffolds to garner further information (Anderson, 2021; Martin et al., 2015; Sullivan, 2019), as is appropriate per sociocultural theory, which states that students learn when provided with scaffolds that allow access to difficult tasks (Steele, 2001). In fact, Anderson (2021) reports that when pressing for reasoning does not occur, a teacher-centered learning structure is taken on rather than an interactive, social structure, which is necessary for learning per sociocultural theory (Steele, 2001).

Varhol et al. (2021) also suggests that through mathematical discourse, students can reach generalizations that allow for stronger mathematical understanding. The researchers discussed the progression through four levels of generalization: *arithmetic generalizations* are based on guessing, whereas algebraic generalizations pass through phases of *factual generalizations* (actions, words, etc.), *contextual generalizations* (use of symbols and language), and *symbolic generalizations* (describe the rule entirely through symbols; Varhol et al., 2021). Symbolic generalization is also the most sophisticated and is the highest level of thinking. In this study, students engaged in mathematical discourse, and the researcher observed their progression through the four levels of generalization (Varhol et al., 2021). The goal of the interactions between the students in this study was to see if students could progress to symbolic generalization, and all three groups of students in the sample population were successful in doing so. The students' thinking became more sophisticated as the conversation continued, even over the course of a single lesson. According to Varhol et al. (2021), the goal of the teacher, therefore, should be to help students progress through these levels of generalization through discourse so that they may participate in a higher level of thinking (Varhol et al., 2021); in other words, the higher level of generalization they reach, the higher level of proficiency attained.

Further, the sociocultural theory implies that students learn through social interactions and the justification of that learning (Allman, 2022; Steele, 2001). In a 2021 study by Ghousseini et al., researchers found that even as early as kindergarten, students benefitted from and engaged in mathematical discourse based on an observation in a kindergarten classroom where a student could justify his reasoning regarding the use of counting by ones and fives to reach 129, which is also supported by the NRC (2001) as a factor promoting mathematical proficiency. This justification can occur in multiple forms (NRC, 2001). The use of gesturing as a developmentally appropriate way for students to communicate (Hynes-Berry et al., 2018) was useful in the study by Ghousseini et al. (2021) when the child was unable to use language to do that justification. Ghousseini et al. (2021) also showed that when the teacher pressed for reasoning, even kindergarteners could justify the mathematics being completed using multiple representations, another standard towards higher proficiency (Collins, 2011; NRC, 2001).

The use of mathematical language, another facet of the sociocultural theory according to Steele (2001), is also important in developing an understanding of mathematics. According to Sigmon et al. (2022), a word sort "enables students to identify shared features of words' meanings and use these characteristics to group words into categories" (p. 8) based on word structure or meaning; sorts can be used at multiple grade levels and are developmentally appropriate for all elementary students. In their analysis of mathematical word sorts for disciplinary vocabulary development and enriched discussion, Sigmon et al. (2022) explained that the word sorts could increase deep understandings of the mathematical content thanks to the level of synthesis that must occur during the sorts Within the study, Sigmon et al. (2022)

identified increased collaboration, problem solving, risk-taking associated with learning goals, and active listening skills, each of which contributes to stronger mathematical proficiency.

Likewise, Smith et al. (2020) claimed that when teachers employed ambitious learning goals related to mathematical discourse regardless of developmental level, those deep understandings of mathematical concepts and increased proficiency would follow. The NRC (2001) corroborates the need for ambitious learning goals for mathematical proficiency. Lastly, Silva (2021) said that students need to be able to use critical thinking to be successful in future careers, which is promoted by mathematical discourse. This is achieved at higher levels, but using mathematical discourse practices throughout their elementary experience increases their ability to use the critical thinking necessary to be successful in 21st century learning (Sigmon et al., 2022; Silva, 2021; Smith et al., 2020).

Discourse and Professional Development

Studies seem to indicate that to help students achieve mathematical proficiency, professional development for teachers is necessary (Martin et al., 2015; Sullivan, 2019). Martin et al. (2015) conducted a study of mathematical discourse at the elementary level. After providing 12 educators with a year-long professional development, the researchers examined the emerging themes regarding mathematical discourse from teachers' instruction. In one example when the teacher used questioning, students were better able to justify their responses, leading to enriched understanding of the content. In another example, questioning led to higher-level critical thinking and evaluation on a multi-step word problem. In a final example specific to questioning, students in first grade were able to develop an understanding of geometric shapes when the teacher used questioning to pull more information (Martin et al., 2015). Each of these instances in the study demonstrated an increased understanding of the mathematical content needed to demonstrate higher proficiency.

Similarly, in a mixed-methods study by Sullivan (2019), participating teachers again underwent professional development to encourage use of mathematical discourse in their middle and high school classes. They found with the help of professional development, the percentage of students engaging increased significantly. In the qualitative portion of this study, the researchers observed a teacher who did not engage in effective discourse with his class, and in doing so, he lowered the rigor of the task (Sullivan, 2019). Based on these two studies (Martin et al., 2015; Sullivan, 2019), when mathematical discourse is used effectively higher rigor can be maintained. Yet to effectively use mathematical discourse, the educator must be prepared with pedagogical strategies that allow them to make instructional decisions to maximize the rich discourse needed to be beneficial to students (Anderson, 2021; Bennett, 2014; Bertolone-Smith & Gillette-Koyen, 2019; Garcia et al., 2021; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Silva, 2021; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021; Wilson & Smith, 2022; Woods, 2021).

Pedagogical Strategies for Facilitating Mathematical Discourse

Many of the above studies (Anderson, 2021; Bertolone-Smith & Gillette-Koyen, 2019; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021) reference various pedagogical strategies that increase the level of mathematical discourse occurring in classrooms. In fact, Martin et al. (2015) found that teacher strategies impact the level of mathematical discourse occurring in the classroom. Questioning, critiquing of student work, establishing clear expectations, thoughtful positioning, scaffolding, and the selection of cognitively demanding tasks are some of the most poignant strategies discussed in existing literature (Anderson, 2021; Bennett, 2014; Bertolone-Smith & Gillette-Koyen, 2019; Garcia et al., 2021; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Silva, 2021; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021; Wilson & Smith, 2022; Woods, 2021). These pedagogical strategies for facilitating mathematical discourse are tightly aligned to NCTM's (2014) mathematical teaching practices, while the use of scaffolding is also based on Vygotsky's (1978) sociocultural theory.

Questioning

Questioning is one such significant strategy that should be used throughout discussions and when engaging with students and their work (Anderson, 2021; Ghousseini et al., 2021; Martin et al., 2015; Smith et al., 2020; Sullivan, 2019). Anderson (2021) examines this strategy in the form of pressing for reasoning in a fourth-grade classroom. In this study, Anderson (2021) observed the participating teacher pressing for reasoning by asking for justification of the response rather than affirming or denying the validity of the answer. In fact, the teacher did this multiple times in a single interaction. By doing so, the teacher elicited further conversation and helped students develop a deeper understanding of both the content and the mathematical language with use of decimals in division, allowing them to be successful in completing their assigned task. Similarly, in the transcriptions from a study completed by Bertolone-Smith and Gillette-Koyen (2019), almost every interaction from the teacher came in the form of a question, apart from only one instance where the teacher summarized the student's response instead. Thie use of questions led to more and higher-level responses from the students.

Smith et al. (2020) describe two types of questions: assessing questions and advancing questions. The assessing questions are used to understand what students are thinking, while advancing questions are used to push students in the direction of their learning goals without providing specific instructions on the path forward (Smith et al., 2020), another form of scaffolding within the ZPD (Steele, 2001). Smith et al. (2020) claim that the purpose of both types of questions is to consider what students might say and to plan in advance of the lesson. They describe a situation in which the teacher in the study prepares advancing questions before the lesson; in doing so, she has something to ask regardless of whether students use an equation to solve the problem in the study or not. This allowed the educator in the study (Smith et al., 2020) to bring the student toward the learning standard even if the student did not make the connection right away. Similarly, in examining the questioning strategies of the teachers in the studies by Anderson (2021) and Bertolone-Smith and Gillette-Koyen (2019), both advancing and assessing questions were utilized; based on the transcripts in both studies, the educators used specific questions to either guide students toward a learning goal or to understand what the student meant. However, it is unclear how intentional the questioning practices of the educators in the studies conducted by Anderson (2021) and Bertolone-Smith and Gillette-Koyen (2019) were.

Critiques of Student Work

Careful and purposeful critiquing of student work is another aspect of pedagogical strategies that teachers must consider when leading mathematical discourse (Anderson, 2021; Bertolone-Smith & Gillette-Koyen, 2019; Ghousseini et al., 2021; Varhol et al., 2021). This is the basis of the third mathematical practice (CCSS.MATH.PRACTICE.MP3), construct viable arguments and critique the reasoning of others, as described within the CCSS (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). Bertolone-Smith and Gillette-Koyen (2019) note that through careful questioning, the teacher can encourage students to analyze the work of others. In a study by Anderson (2021), for example, the participating teacher asked students if they agreed or disagreed with responses given by a student. In one instance, the teacher asked if a student, MJ, agreed or disagreed with another student's strategy. MJ responded that he did but that he had used a different strategy, which led to a discussion about that strategy. Similarly, Ghousseini et al. (2021) observed that when the teacher asked pressing questions about one student's work, it enabled other students to begin assessing that work, interpret it, and respond to it, as well. In doing so, the onus for learning and critiquing fell upon the students rather than the teacher (Ghousseini et al., 2021).

Varhol et al. (2021) identifies eight types of interactions to include getting in contact, locating, identifying, advocating, thinking aloud, reformulating, challenging, and evaluating, and discovered connections between which ones led to higher levels of generalization (advocating, locating, and reformulating) in their study. In fact, both advocating and reformulating are reminiscent of NCTM's (2014) standard of critiquing other's work, which NCTM argues is a necessary skill in mathematics. Teachers must provide opportunities for students to be able to engage in those types of interactions and possibly teach students how to interact that way (Varhol et al., 2021). This study provides evidence that being able to cooperate with others and build upon the work and thoughts of others is important in making higher level generalizations, which is necessary to achieve higher mathematical proficiency, as well.

Establishing Clear Expectations

The ability for the students in the above studies (Anderson, 2021; Bertolone-Smith & Gillette-Koyen, 2019; Ghousseini et al., 2021; Varhol et al., 2021) to critique the work of others did not happen on its own, however (Bennett, 2014; Ghousseini et al., 2021; Smith, 2021; Smith et al., 2020). Importantly, one key characteristic of successful mathematical discourse is the establishment of clear expectations from the onset (Bennett, 2014; Ghousseini et al., 2021;

Smith, 2021; Smith et al., 2020). In their observation of the teacher in their study, Ghousseini et al. (2021) noticed that students had the expectation that they must explain their thinking; they also had the expectation that they could use resources around the room if they needed them. The resources could be used at the student's discretion or be suggested by the teacher. So, to garner stronger responses from questioning and to encourage the critiquing of other's work in the Ghousseini et al. (2021) study, expectations had to be in place first.

Bennett (2014) takes the need for expectations further and suggests that for discourse to occur at all in a classroom, there must be a culture of participation. In a study of thirteen classrooms, Bennett (2014) found three ways that this culture arose, including the creation of classroom norms, a set of classroom procedures for participation, and the diversification of the discourse. The first strategy, classroom norms, sets the expectation that everyone will participate. One way of doing this is to allow students to help create the classroom rules. Another way is setting up the desks in small groups, thus creating a structure that encourages discussion. The second strategy, classroom procedures, built upon procedures that are common in many classrooms, including the utilization of "wait time, calling on different students, or consequences for not participating" (Bennett, 2014, p. 22). For example, in one classroom, the teacher used a certain number of raised hands to determine whether students were ready; so, the marker of hands indicated the amount of wait time needed. Lastly, teachers provided opportunities to diversify the discourse, including varying the groups for discussions or use of different opportunities to discuss such as number talks or discussions of solutions. Bennett (2014) noticed that the positive culture created by the classroom procedures and norms need to be in place for discussion to take place.

Smith (2021), on the other hand, analyzed the use of individual expectations in the form of individual goals rather than class expectations as Ghousseini et al. (2021) and Bennett (2014) suggest. Students in the study by Smith (2021) had mathematics goals, language goals, and social-emotional goals (Smith, 2021). The goals could be written for the class as a whole, but teachers in the study established expectations for mathematical discourse that allowed access by all students individually. Smith et al. (2020) also describes goal setting as a necessary component before any mathematical discourse can occur, which supports Bennett's (2014) finding of the establishment of norms and procedures before discourse, whether independent or whole class.

Scaffolding

For students to be successfully in the zone of proximal development (ZPD) as described by Allman (2022) and to reach their mathematics, language, or social-emotional goals, the teacher must have in place scaffolds that support students on a variety of levels. Among the most useful scaffolds for mathematics learning identified in existing studies were anchor charts (Bertolone-Smith & Gillette-Koyen, 2019; Garcia et al., 2021), visual cues (Smith et al., 2020; Wilson & Smith, 2022; Woods, 2021), sentence frames or word guides (Sigmon et al., 2022; Wilson & Smith, 2022), and integration of student choice (Silva, 2021; Sullivan, 2019). Bertolone-Smith and Gillette-Koyen (2019) suggest using anchor charts, or classroom displays that show the class's collective understanding of mathematics to which students can then refer back throughout discussions. Garcia et al. (2021) compared less skillful and more skillful recordings for anchor charts, noting four elements of more skillful recordings: the anchor charts should be accurate representations of what the class or student described, the mathematics itself should be accurate, the recordings are visible and organized, and the representations relate to what students will use. In fact, when the students' thinking was visible with anchor charts, preservice teachers in the Woods (2021) study noticed that students could expand upon and revise their thinking, which allowed for an expansion of the mathematics goal for the lesson. For students to get that information into sentences or phrases that can be used on anchor charts, sentence frames (example ways to start or word sentences) and word guides (vocabulary translations for other languages) are especially useful in supporting students who may need help accessing the language (Sigmon et al., 2022; Wilson & Smith, 2022). This will be important in understanding mathematical language development, as well (Sigmon et al., 2022; Wilson & Smith, 2022).

Another scaffolding strategy that can be used is student choice, which refers to the ability of students to select how they approach a problem (Silva, 2021; Sullivan, 2019). Silva (2021) and Sullivan (2019) both encourage choice to support students who need different ways to access discourse. In a study by Silva (2021), English Language Learners (ELLs) were able to choose the language in which they spoke, the strategies they used, whether they worked alone or with others, and how they participated. When these students could choose which language they used, they were able to better concentrate on the mathematics content rather than the demands of the English language. Students in this study could also participate silently and show their work visually, which is still an aspect of mathematical discourse (Silva, 2021), or they could describe their work in their chosen language. When they used their choice of strategy throughout the discussion, they were able to not only better describe their work but could attend to the strategies others used, too. Students in Silva's (2021) study preferred to work with others, as well. Two students who worked together were able to use what their partners did to construct their own understanding and were more successful working together (Silva, 2021). Sullivan (2019), on the other hand, found that choice led to increased mathematical authority, where students led the discussion in the lesson, which may benefit students by increasing mathematical discourse (Bennett, 2014; Smith, 2021; Sullivan, 2019; Wilson & Smith, 2022; Woods, 2021). Neither study (Silva, 2021; Sullivan, 2019) addressed what happens when choice cannot occur or when the language is inaccessible to the teacher.

Thoughtful Positioning

Further, thoughtful positioning as a pedagogical practice also improves mathematical discourse (Bennett, 2014; Smith, 2021; Sullivan, 2019; Wilson & Smith, 2022; Woods, 2021). Research from Bennett (2014), Smith (2021), Sullivan (2019), Wilson and Smith (2022), and Woods (2021) each claim that learning should be student-centered, where students are positioned in a way that they are in control of their learning and the discussion and where mathematical autonomy is gained. In a study by Bennett (2014) of best practices for discourse, it was found that students need to be positioned in a way that they take ownership of their participation. He writes that students in classrooms with thoughtful positioning "recapture the intellectual authority of the classroom that is rightfully theirs" (Bennett, 2014, p. 24) and creates a rich learning environment. Sullivan (2019) corroborates this finding. In a study by Sullivan (2019) of teacher beliefs regarding discourse, the level of discourse, and the mathematical authority given to students during discourse, the class in which the teacher gave greater authority to students saw increased mathematical discourse compared to the class in which the teacher shared little authority with the students. Woods (2021) similarly noticed that positioning of students as "sense makers" (p. 781), or those in charge of making sense of the mathematics rather than absorbing knowledge from the teachers, led to more correct answers in their work.

Smith (2021) asserts that when students are placed in a position where they are more likely to be involved actively in conversation and feel valued as a member of the mathematical community, they are more likely to engage in discourse. Wilson and Smith (2022) similarly note that students struggling with the social expectations of the discourse need to be supported rather than avoided during mathematical discourse. A common mistake teachers make, that Wilson and Smith (2022) argue, is avoiding calling on or decontextualizing the problem for students who may not have the language or content skills yet to tackle the challenging tasks (i.e., the students who are still in the "too difficult" section of the ZPD spectrum); to do so is a detriment to their learning. Rather, they claim, teachers should make careful instructional decisions to hold these students accountable via positioning and put supports in place to help them access the discourse and participate actively (Wilson and Smith, 2022).

In her beginning of school year message to mathematics educators, NCTM president Trina Wilkerson (2022) reiterated the importance of positioning. She describes three types of positions: student-student, teacher-student, and mathematics. Positioning students as mathematicians and building their confidence through student-student discourse is important not only in mathematics, Wilkerson (2022) claims, but just as a best practice in education. With teacher-student positioning, the teacher must be willing to relinquish control and allow students to feel competent and have authority over mathematics. Overall, though, Wilkerson (2022) describes it as a partnered effort, where teachers and students engage together in mathematical discourse. Lastly, she states that educators should be positioning mathematics in a way that demonstrates its importance through the selection of high-quality tasks in which students are engaged deeply in their work and discussions (Wilkerson, 2022). When these three positions are taken, educators empower students to "contribute and lead" (para. 14) not only in school, but in society as well.

Cognitively Demanding Tasks

Existing research supports Wilkerson's (2022) claim that mathematics can be positioned strongly by using high quality, or cognitively demanding, tasks (Martin et al., 2015; Smith et al., 2020; Wilson & Smith, 2022). Mathematical discourse is richer for students when presented with cognitively demanding tasks (Martin et al., 2021). In mathematics, higher-level or cognitively demanding tasks either require the making of connections between concepts or the exploration of concepts that require significant cognitive effort (NCTM, 2014). In fact, the use of cognitively demanding tasks is the backbone of mathematical discourse, as Smith et al. (2020) argues effective discourse cannot happen without rich tasks. As such, Martin et al. (2015) found that by using cognitively demanding tasks, students were able to engage in "more efficient and effective ways of problem solving" (p. 17). Cognitively demanding tasks also increased the amount of mathematical discourse occurring in the observed classroom in the study by Martin et al. (2015).

Smith et al. (2020) describe a few ways in which teachers can select a cognitively demanding task. They suggest adapting existing tasks from low-level tasks typically found in textbooks, using another resource other than a school's adopted textbook to find a task, or creating a task independently. However, in locating these tasks, it is vital that the teacher ensures that the task is aligned with the learning goals for the lesson. According to Smith et al. (2020), without alignment between the goal and the task, the teacher cannot adequately prepare for student responses and identify the necessary strategies for learning. Yet many teachers are tied to a resource that must be used with fidelity (NRC, 2001). Smith et al. (2020) do not seem to account for this.

As with the expectation for mathematical language development, tasks should not be made easier for students who cannot attend to the level of cognitive demand; rather, scaffolds should be put in place to support students who cannot access the learning as is (Wilson & Smith, 2022). Scaffolds assist teachers in supporting students who may not be able to attend to the level of cognitive demand (Iris Center, 2022). Teachers should also keep in mind the developmental levels of their students, especially as described by sociocultural theory. However, Martin et al., (2015), Smith et al. (2020), and Wilson & Smith (2022) fail to explain how teachers decide what tasks to select and whether they take student needs into consideration when selecting these tasks.

Mathematical Language Development

Mathematical discourse results in an increase in disciplinary language use (Croce & McCormick, 2019; Sigmon et al., 2022; Xu & Clarke, 2019). Sigmon et al. (2022) write, “Disciplinary literacy starts from the idea that academic disciplines have particular ways of constructing, synthesizing, and assessing knowledge” (p. 6). This construction, synthesis, and assessment is needed to improve students’ mathematical proficiency (Sigmon et al., 2022). Students need to be able to use mathematical language to talk effectively about the math they use (Sigmon et al., 2022). Croce and McCormick (2019) further explain that mathematical language is necessary to understand and then solve problems. Sigmon et al. (2022) also purport that use of mathematical language in discussion increases mathematical proficiency.

Xu and Clarke (2019) specifically referenced Vygotsky’s (1978) sociocultural theory in their study, claiming that learning is a social process and forms the basis of their study. The study assessed the use of mathematical discourse as a form of learning in primary schools in Shanghai, Seoul, and Melbourne. They claimed that the sociocultural framework explained the need for discourse in mathematics education, even when culture varies. While mathematical discourse can look different in other countries compared to the United States, the researchers found that it still existed thoroughly (Xu & Clarke, 2019). In countries like South Korea and

China, choral responses were the norm compared to Australia, where individual responses were more common, as is the case in the United States. However, while the classrooms in Melbourne were considered interactive and student-centered, the use of mathematical language was lower than in the other classrooms of Shanghai and Seoul. The choral responses, in fact, led to greater mathematical language use in South Korea and China (Xu & Clarke, 2019).

In considering the mathematical language of tasks, especially cognitively demanding tasks, it is a common mistake to remove the context or underestimate students who may not be able to access the language as is (Ghousseini et al., 2021; Silva, 2021; Wilson & Smith, 2022). Instead, teachers should implement the scaffolds mentioned above rather than decontextualize the problems or remove the mathematical language altogether (Silva, 2021; Wilson & Smith, 2022). The tendency to decontextualize disadvantages elementary students further (Wilson & Smith, 2022). Instead, it may be pertinent to allow students to respond in a variety of ways (numerically, symbolically, verbally, or graphically), where they may still demonstrate their understanding of the language in a way more accessible to them (NRC, 2001).

Summary

Mathematical discourse is a necessary component of high-quality mathematical classroom instruction and learning according to NCTM (2014). Overall, the existing research indicates that mathematical discourse in fact leads to greater mathematical proficiency (Anderson, 2021; Bertolone-Smith & Gillette-Koyen, 2019; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021), where mathematical proficiency is defined by five interconnected strands (Collins, 2011; Corrêa & Haslam, 2021; Groves, 2012; NCTM, 2021; NRC, 2001; Suh, 2007). Further, research-based strategies that have shown to increase discourse such as the ones found in the existing literature

(Anderson, 2021; Bennett, 2014; Bertolone-Smith & Gillette-Koyen, 2019; Garcia et al., 2021; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Silva, 2021; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021; Wilson & Smith, 2022; Woods, 2021) are key features that will be examined in the present phenomenological study. Lastly, the use and development of mathematical language through mathematical discourse, which is associated with increased proficiency (Croce & McCormick, 2019; Sigmon et al., 2022; Wilson & Smith, 2022; Xu & Clarke, 2019), must also be examined for further evidence of use by educators. Based on the existing research, more information is needed to understand what is happening in classrooms across the United States, especially as mathematical proficiency is failing to improve (OECD, 2019). Based on this evidence, the purpose of this qualitative phenomenological study was to explore the perceptions of public elementary K-5 educators regarding the use of mathematical discourse as a pedagogical strategy, where mathematical discourse is defined by NCTM (2010) as "the mathematical communication that occurs in a classroom" (p. 1).

CHAPTER 3: METHODOLOGY

Mathematical discourse as a pedagogical tool recently garnered interest from researchers due in part to the reform work led by the National Council of Teachers of Mathematics (NCTM; 2000, 2014). The reforms suggested in *Principles and Standards for School Mathematics* (2000) indicated the need for mathematical communication, with the Common Core State Standards (CCSS; 2010) introducing mathematical communication as a necessary skill for students. In NCTM's *Principles to Actions: Ensuring Mathematical Success for All* (2014), teachers were encouraged to promote mathematical communication, now called mathematical discourse, as part of the mathematics teaching practices. However, even with the reform proposed over the past few decades (NCTM, 2000; NCTM, 2014), mathematical proficiency stagnated (OECD, 2019) and then decreased following the COVID-19 pandemic (NCES, 2022a).

This chapter defines the site information and participant sample, the research design, the data collection and analysis methods, and the limitations and ethical considerations of the study. Although use of mathematical discourse is an expectation in all public schools (K-12), the focus of this study was on the experiences of public elementary K-5 school educators. A phenomenological approach was used to explore these experiences and to learn about the perceptions of elementary teachers. The purpose of this qualitative phenomenological study was to explore the perceptions of public elementary K-5 educators regarding the use of mathematical discourse as a pedagogical strategy, where mathematical discourse is defined by NCTM (2010) as "the mathematical communication that occurs in a classroom" (p. 1). This purpose was based on the evidence that progress is not being made in mathematics in the United States, as evidenced by the Programme for International Student Assessment (PISA) and the National Assessment for Educational Progress (NAEP) taken within the past 5 years (NCES, 2022a;

OECD, 2019). This, partnered with evidence that mathematical discourse leads to increased mathematical proficiency (Anderson, 2021; Bertolone-Smith & Gillette-Koyen, 2019; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021), could indicate that there is a lack of usage of mathematical discourse by elementary educators. Thus, it was necessary to explore how educators perceive mathematical discourse. To achieve this, the following research questions were used:

Main Research Question. What are the perceptions of public elementary K-5 teachers regarding the use of mathematical discourse as a pedagogical strategy in the classroom?

> **Sub-Question 1**. How do public elementary teachers describe how they use different pedagogical strategies for facilitating mathematical discourse?
>
> **Sub-Question 2**. How do teachers describe how students engage in mathematical discourse while using disciplinary language?

To explore these perceptions of public elementary educators regarding the use of mathematical discourse as a pedagogical strategy, a phenomenological approach was taken. In this study, public school educators shared their lived experiences to explain the phenomenon of the use of mathematical discourse as a pedagogical strategy.

Creswell and Creswell (2018) describe several characteristics of qualitative research, which describe the site, types of data, and data analysis processes. Each of these characteristics shape the present study and its design. For example, by using semi-structured interviews, the design both allows for the open-ended design and the emergent design structure. These are described in greater detail within this chapter.

Site Information and Demographics

The sites used for the study were 10 public schools located in Connecticut, Maine, New Hampshire, Rhode Island, and Vermont, each of which utilize the Common Core State Standards (CCSS). The reason for this scope of states is that the CCSS includes the expectation that students partake in mathematical discourse (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). All New England states utilize the CCSS as the framework for their standards, excluding Massachusetts, which uses a modified version ("United States Standards," 2023). Therefore, state standards in the remaining states require educators to facilitate mathematical discourse (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010).

Because phenomenological research looks deeply at the lived experiences of participants (Bloomberg & Volpe, 2019), between six and 12 participants typically are used (Moser & Korstjens, 2018). Thus, a limit was set on the number of sites: two sites from each state were selected for participant recruitment to ensure consistent representation of the New England states. Sites were selected based on two factors: school population and demographics. The student population size of each site was based on the average population of elementary schools in the represented state. The demographics were also similar related to racial diversity of students. The diversity of the schools parallels the average diversity of the states.

Ten elementary schools were identified as potential sites of study based on school population and demographics. Statistics for the schools are current as of 2022 and are publicly available from the National Center for Education Statistics (NCES). The population of each school is representative of the average primary school size for schools in the given state: 424 for Connecticut, 270 for Maine, 308 for New Hampshire, 394 for Rhode Island, and 239 for

Vermont (NCES, 2020). The school populations are each close to the average population, but more emphasis was placed on the demographics being representative of the state as described by the U.S. Census information (U.S. Census Bureau, 2022a; U.S. Census Bureau, 2022b; U.S. Census Bureau, 2022c; U.S. Census Bureau, 2022d; U.S. Census Bureau, 2022e). Potential participants were recruited from these schools until 10 participants who met the criteria of participation were reached. To ensure confidentiality, the schools were not identified.

Participants and Sampling Method

This section includes information about the participants in the study and how they were selected. Ravitch and Carl (2021) indicate that purposeful sampling is used in qualitative research, which means in phenomenological that the researcher will choose specific participants that meet the criteria of the phenomenon being studied. This ensures that participants have information relevant to the study. In this study, participants were selected who were public school elementary (with sites focused on K-5) teachers who taught math within the identified schools to a general education population. Participant criteria included: age 18 or older; currently employed at a public elementary school; at least one year teaching mathematics at their current grade level to ensure some familiarity with the curriculum; and teaches grades K-5. Information collected in the master list included: the educator's grade level, school affiliation, school e-mail address, and years of teaching experience.

Each of the study sites described in the section above are public elementary schools. Educator email addresses are publicly available information and were accessed from the school websites. The researcher sent out a recruitment email to the general education staff at each school. The participants self-identified as meeting the recruitment criteria and reached out via email to the researcher. The goal was to have a representative sample of teachers K-5, with at least one teacher at each of these grade levels, but participants included one kindergarten teacher, one first grade teacher, two third grade teachers, five fourth grade teachers, and one fifth grade teacher. Emails were sent out once per week for 5 weeks until 10 participants volunteered for the study. The email, available in Appendix B, asked educators to participate in one 30 to 60-minute interview via Zoom to explore perceptions of the use of mathematical discourse as a pedagogical strategy. Emails were sent through the principal investigator's University of New England (UNE) email address, and the participant information sheet (Appendix C) was included as an attachment. Interested participants who self-identified as meeting the criteria were asked to reach out to the principal investigator through her UNE email, which was provided on the initial recruitment email and the participant information sheet. The participant information sheet was reviewed at the start of the interview. After reviewing the participant information sheet, participants were asked if they wished to continue and had an opportunity to discuss any questions or concerns before the interview began. The participant then acknowledged verbally if they would like to proceed with the interview. Participants also confirmed their consent for audio recording once more prior to the start of the interview.

Instrumentation and Data Collection

Creswell and Creswell (2018) indicate that the researcher is the instrument through which the study is conducted. This section describes how the process occurred through the data collection, including every step of the dissertation process to this point. This section also includes information on the member checks. As phenomenological research typically involves interviews to learn about the lived experiences of the participants (Creswell & Creswell, 2018), the present study used a semi-structured interview approach. Ravitch and Carl (2021) explain that the semi-structured interview uses the same questions for all participants, but the order, wording, and the probing may look different for each participant. Structured and unstructured interviews can be used and have their own benefits; structured interviews allow for stronger comparisons of information, while unstructured interviews allow for more in-depth and personal responses (Leavy, 2014). However, Leavy (2014) argues that structured interviews fail to examine social life beyond the cultural expectations of how to respond to questions, while unstructured interviews are better used for narrative designs. Semi-structured interviews are the most often used structure for qualitative research and can better guide the participant towards discussion of research-relevant topics (Leavy, 2014).

The questions for this semi-structured interview were created based on the work of Castillo-Montoya (2016). The interview protocol for this study and the alignment for these interview questions, based specifically on the research questions, can be reviewed in Appendix D. The qualitative study was conducted virtually using Zoom and recorded for audio. Every participant was asked the same questions but prompting differed based on the responses from a given participant. Every participant was provided with a pseudonym and was given a school identifier number to maintain their confidentiality. The master list was saved on a password protected USB and then destroyed following the conclusion of the interviews and transcript checks. Following the transcription of the interviews, participants were given the opportunity to review their transcripts for accuracy. The transcripts were emailed to the participants, and they were given 5 business days to respond. If no response was received within this period, the principal investigator assumed the participant had no comments, and the transcript was assumed to be accurate. Participants were able to make clarifying remarks, redact information, or confirm the transcript for accuracy. Once the transcript check was completed, the master list was deleted from the password protected USB and all identifiable information was destroyed.

Data Analysis

This section explains in greater detail the data analysis process. The data analysis process for qualitative data includes six steps identified by Creswell and Guetterman (2019): (1) preparing and organizing the data for analysis; (2) initial exploration of the data through the process of coding; (3) using codes to develop descriptions and themes; (4) representing the findings through narratives and visuals; (5) interpreting the meaning of the results through personal reflection and use of literature; and (6) conducting strategies to validate the accuracy of the findings. Each of these steps will be explained in full detail relevant to the present study.

This data analysis begins with the transcription and continues with the initial coding of that transcript. Ravitch and Carl (2021) explain that data organization allows the researcher to identify patterns and relationships among the data. These patterns and relationships can help identify the themes from the research, which can then be used to analyze the data through a critical lens. The transcription process is viewed as the initial step of analysis (Ravitch & Carl, 2021). Several considerations must be made in the transcription process. First, the interviews were conducted via Zoom and recorded for audio using the program. Following the interviews with the participants, the researcher used the auto-transcribe feature within Zoom and then confirmed each transcription for accuracy and consistency by listening to the audio. Participants were emailed copies of the transcript to their provided email addresses and were asked to review it for accuracy. Once the data was transcribed and reviewed for accuracy, the rest of the data analysis continued with coding.

Precoding was used initially to become more familiar with the transcript. Precoding occurs before the formal coding process and involves reading and marking the data with first impressions or takeaways (Ravitch & Carl, 2021). The precoding occurred within Atlas.ti and involved highlighting the data without leaving specific codes. After this, multiple readings of the transcripts were conducted, each with different purposes. These purposes included reading based specifically on the research questions, reading pieces of each transcript for patterns, and reading for the specific theories used in the conceptual and theoretical frameworks. The transcripts were coded using Atlas.ti to identify patterns and themes in the text (Ravitch & Carl, 2021). Both in vivo and deductive codes were used during the coding process. Ravitch and Carl (2021) describe the coding process as both inductive and deductive; in vivo codes are inductive, where the participants words are used for the codes, while deductive codes come from interpretations of the participants words as related to the literature. Atlas.ti was used to manually code each transcript and then organize the codes into code sets, where the codes were then clustered and analyzed for themes. Features used with Atlas.ti include the document manager, wherein the transcripts were manually coded using highlighting and tagging functions; quotation manager, wherein the researcher reviewed participant quotes from different transcripts with the same codes; and the code manager, wherein codes were organized, deleted, or renamed.

Limitations, Delimitations, and Ethical Issues

This section discusses any conflicts of interest, including the researcher's own connection to the topic. The researcher is a New Hampshire public school math teacher who actively uses mathematical discourse in her class, so there is the possibility that her own understanding can bias the results. Because some of the participants may be from New Hampshire, there may be some biases because the researcher is also a New Hampshire teacher. The researcher is also on the board of the New Hampshire Teachers of Mathematics, so this could impact the results, as well, both in the form of bias and if the participants are aware. Ravitch and Carl (2021) also discuss the power asymmetry that comes with interpreting the experiences of others. They claim that the researcher of any given study makes assumptions that influence the interpretations of the data. Therefore, it is necessary to be transparent regarding these existing assumptions. The researcher assumes that each participant does, in fact, use mathematical discourse.

This section also provides information about the protection of the participants (i.e., anonymity) and what consequences there may be of participating in the study. Pseudonyms were used for each participant. The sites are only associated with the participants as School 1, 2, 3, etc. Because the participants are described using their grade level, the teacher could be revealed if in a small school without the use of pseudonyms and the renaming of schools. Participants should not be negatively affected by the study.

Limitations

The researcher was transparent with what limitations exist for a study and must address them to the best of their ability, as well (Roberts & Hyatt, 2019). Roberts and Hyatt (2019) define limitations as those features of a study that "may affect the results or your ability to generalize the findings" (p. 154). These are often limitations, or flaws, because of the methodology. In this case, phenomenological research utilizes a small sample size and cannot be generalized to larger populations or contexts. Further, the background (such as culture, years teaching, etc.) of each participant may have influenced the results and led to inconsistencies.

Another limitation was in the recruitment of the participants. The participants who volunteered for the study may have done so because of their strong feelings surrounding mathematical discourse. Educators who perceived mathematical discourse neutrally might not have volunteered to participate, swaying the results either positively or negatively. Additionally, recruitment occurred near the end of the school year or the beginning of the summer for teachers.

Participants who might have otherwise may have been dissuaded from volunteering due to the demands of the end of the school year or being on vacation.

Delimitations

Delimitations describe the choices made by the researcher to narrow the scope of the study (Bloomberg & Volpe, 2019). This study focused on the lived experiences of public elementary educators within the participating sites, and thus the participants are limited to volunteers from 10 schools. Not all schools in the United States, nor within the states identified, were invited to participate in the study. The reason for this was to ensure the participating sites utilized the Common Core and represented the average demographics and populations of the respective state. This study sought participants from schools in locations that were expected to facilitate mathematical discourse to explore perceptions of teachers. Additionally, by not asking every school to participate, a full picture of teacher perceptions may not be understood in its entirety. Additionally, the public schools selected as sites for recruitment were in states that utilize the Common Core State Standards (CCSS). This ensured that educators participating in the study do, in fact, utilize mathematical discourse in their classrooms, as this is an expectation of the CCSS (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010).

The participants also were limited purposefully to those who taught mathematics in an elementary school serving grades kindergarten through 5th and had at least one year of experience at their current grade level. By requiring these educators to have at least one year of experience at their current grade level, the researcher could ensure participants could describe their pedagogy and their facilitation of mathematical discourse with at least one full year of experience to support their claims. The participants had a full understanding of the standards and curriculum for their grade levels, as well, by having at least one year of experience at their grade level. The participants were also general education teachers. Special educators or curriculum specialists also could explain their experiences regarding mathematical discourse, but the scope of this study was limited to the experiences of teachers of the general population to get an overall understanding of teacher perceptions.

Ethical Issues

The Belmont Report from the National Commission for the Protection of Human Subjects of Biomedical and Behavioral Research (1979) provides guidelines for ethical research with three main parts: the setting of boundaries between practice and research, basic ethical principles, and applications. Part B on the basic ethical principles states that participants need to "enter into the research voluntarily and with adequate information" (National Commission for the Protection of Human Subjects of Biomedical and Behavioral Research, 1979, p. 4). To do this, participants must provide informed consent (Part C), wherein the participants are provided with information about the study, including potential risk, the researcher ensures that the participants understand the information provided, and the participants voluntarily comply with participation in the study (National Commission for the Protection of Human Subjects of Biomedical and Behavioral Research, 1979). Throughout the study, the researcher must also maintain the safety of the participants; participants cannot be harmed, and the benefits of the study for the participants must be maximized while the risks are minimized (National Commission for the Protection of Human Subjects of Biomedical and Behavioral Research, 1979). Those benefits must be equally distributed based on need, effort, societal contribution, and merit, while the risks must be necessary to conduct the research (National Commission for the Protection of Human Subjects of Biomedical and Behavioral Research, 1979). Lastly, the

National Commission for the Protection of Human Subjects of Biomedical and Behavioral Research (1979) speaks to the selection of participants, stating that "there be fair procedures and outcomes in the selection of research subjects" (p. 9).

Given this information, this section addresses the informed consent process and confirms that participants volunteered to participate and understood both the risk and benefits of the study. After the email sent by the researcher from her University of New England email address to potential participants, prospective participants emailed the researcher back at the same email address. Participants were volunteers and were not financially compensated. All participants provided informed consent at the start of the interview after being provided with a participant information sheet that detailed the study, its purpose, and any risks or benefits associated with participation. The participant information sheet can be found in Appendix C.

Further the storage and disposal of the data must also be addressed. Data was not stored on any cloud-connected software, and paper copies of work associated with the study were locked in a file cabinet only accessible by the researcher. Digital copies were stored on a password-protected USB drive. A backup USB drive, also password-protected, stored the data as well and was locked in a secured file cabinet. Once the objectives of the study were completed and after a period of 3 years, the data will be destroyed: paper copies will be shredded, and digital copies on the USB will be permanently deleted.

Trustworthiness

This section summarizes the following sub-categories: credibility, transferability, dependability/validity, and confirmability. Bloomberg and Volpe (2019) note that transparency of how a researcher ensures trustworthiness is a critical aspect of research. Trustworthiness requires that the researcher controls biases (Bloomberg & Volpe, 2019). The purpose of this section is to describe how the data from this study can be trusted.

Credibility

Credibility refers to the accurate representation of the data from all involved parties (Bloomberg & Volpe, 2019). One way to ensure credibility is with the use of participant or member checks (Bloomberg & Volpe, 2019). Participants reviewed the transcripts for accuracy. Following the transcription of the interview, the researcher reviewed the accuracy of each transcript with the participants to ensure that everything in the transcript was represented fully and accurately. Transcripts were emailed to the participants individually upon completion, and participants were given a period of 5 business days to review the transcripts. If no response was given within this period, the transcript was assumed to be accurate.

Transferability

Transferability refers to the ability to apply the findings of a study to broader contexts (Bloomberg & Volpe, 2019). This section identifies what evidence was used to show that the work done in a qualitative context can apply to other settings. In qualitative research, the results are not assumed to be generalizable (Bloomberg & Volpe, 2019). Instead, Bloomberg and Volpe (2019) claim that transferability should allow readers to "decide whether similar processes will be at work in their own settings" (p. 205).

Transferability is assessed by reviewing the purposeful sampling of the participants from public elementary schools in New England and utilizing thick description. Public schools in New England (excluding Massachusetts) were selected to recruit participants from based on their population and demographics as representations of average state data. These schools were identified using publicly available information. General educators at each school were sent a recruitment email (Appendix B) using publicly available information from the school websites. Participants self-identified as meeting the recruitment criteria and reached out to the researcher. Interviews were conducted via Zoom using a premium account, and transcripts were coded using Atlas.ti, a paid program used for qualitative analysis. Appendices B, C, and D provide the recruitment email, participant information sheet, and interview protocol respectively, each of which can be replicated in a future study and applied to new settings. Each of these factors indicates the transferability of this study.

Dependability and Validity

Ravitch and Carl (2021) discuss eight strategies to ensure validity in qualitative studies, which include triangulation, participant validation, strategic sequencing of methods, thick description, dialogic engagement, multiple coding, structured reflexivity processes, and disconfirming evidence. This study used dialogic engagement, participant validation, thick description, and structured reflexivity processes. Dialogic engagement occurred with conducting member checks. Transcript validation served as participant validation and occurred after the transcription. Thick descriptions of the data are provided in the data analysis portion of this report. Structured reflexivity processes were used during the data analysis.

Dependability can be achieved by ensuring that “the research process is clearly documented, logical, and traceable” (Bloomberg & Volpe, 2019). Triangulation, as described above, can increase dependability (Bloomberg & Volpe, 2019). The researcher ensured inter-rater reliability by asking the lead advisor to review the process of coding, the emergent themes, and the conclusions made. The data was compared for differences and similarities and then further interpreted to address any differences.

Confirmability

Confirmability refers to the idea that it is impossible to be entirely objective and instead stay relatively neutral; if someone else were to perform the study, the researcher could find similar results even with a different set of biases (Ravitch & Carl, 2021). Ravitch and Carl (2021) describe four ways to demonstrate reflexivity, including researcher memos, a research journal, dialogic engagement, and researcher interviews. In other words, there should be some sort of trail indicating reflexivity. While it is not possible to be totally objective, it is possible to be clear about the processes used throughout the study (Bloomberg & Volpe, 2019).

Summary

This chapter provided a description of the methodology of this qualitative phenomenological study. Semi-structured interviews were used with participants from 10 schools in Connecticut, Maine, New Hampshire, Rhode Island, and Vermont. These interviews were transcribed using speech-to-text software embedded within Zoom, and Atlas.ti software was used to code the transcripts. The participants checked the transcripts to ensure that they were accurate. Participants had their confidentiality protected with pseudonyms, and potential conflicts of interest were addressed ahead of time, as well. Participants were provided with a participant information sheet (Appendix C) and were assured they would not be met with any negative consequences by participating in the study. The researcher ensured trustworthiness by building credibility, validity, and dependability. The methodology and results are transferable and confirmable. The methodology described in this chapter provides the foundation through which an exploration of the perceptions of the use of mathematical discourse by public elementary K-5 teachers can be made. The next chapter presents the data collected in the study.

CHAPTER 4: RESULTS

The purpose of this study was to explore the perceptions of public elementary K-5 educators regarding use of mathematical discourse as a pedagogical strategy, where mathematical discourse is defined by NCTM (2010) as "the mathematical communication that occurs in a classroom" (p. 1). The research question and sub-research questions are as follows:

Main Research Question. What are the perceptions of public elementary K-5 teachers regarding the use of mathematical discourse as a pedagogical strategy in the classroom?

Sub-Question 1. How do public elementary teachers describe how they use different pedagogical strategies for facilitating mathematical discourse?

Sub-Question 2. How do teachers describe how students engage in mathematical discourse while using disciplinary language?

Following approval of the study by the Institutional Review Board (IRB), participant recruitment began. Participants were recruited from 10 specific schools, which were selected because they are in New England states that utilize Common Core (Connecticut, Maine, New Hampshire, Rhode Island, and Vermont) and were generally representative of the state's demographics. Recruitment emails (Appendix B) were sent to all classroom educators at the school, but additional criteria for participation included: (a) were age 18 or older; (b) were currently employed at a public elementary school; (c) had at least one year teaching mathematics at their current grade level to ensure some familiarity with the curriculum; and (d) taught in grades K-5. The potential participants were asked to self-identify as meeting these criteria. The recruitment email was sent to the educators at these schools five times until a total of 12 educators responded to the recruitment email. None of the participants were known previously to the researcher.

Of the 12 educators who responded to the recruitment email, 11 educators moved through to the interview stage of the study. However, during the interview stage, one of the educators had a change of employment and no longer met the inclusion criteria of teaching in a K-5 school. The interview was conducted, but after being informed of this change, the data for this participant was deleted. A total of 10 viable participants were ultimately used. Each participant was assigned a pseudonym for the study. An individual 30–60-minute semi-structured interview was scheduled for each participant. By using a semi-structured interview format, all participants were asked the same open-ended questions, but it also allowed the researcher to ask follow-up questions based on the responses of the participants when needed.

The 10 interviews were audio-recorded in Zoom with permission from the participants. The interviews were transcribed using Zoom's auto-transcription service. The researcher read through and cleaned up each transcript to ensure accuracy based on the audio recording. The transcripts were sent to each participant for validation. Each participant was given 5 business days to review the transcripts and make any revisions. Only one participant requested corrections, which were made. Following these corrections, all 10 transcripts were accepted, and the master list with identifiable information was deleted. With the transcripts complete and the master list destroyed, the data collection portion of the study concluded.

Analysis Method

The goal of this study was to understand the perceptions of public elementary teachers regarding their use of mathematical discourse as a pedagogical strategy. To achieve this goal, this study utilized the six steps described by Creswell and Guetterman (2019) for data analysis, including (1) preparing and organizing the data for analysis; (2) initial exploration of the data through the process of coding; (3) using codes to develop descriptions and themes; (4)

representing the findings through narratives and visuals; (5) interpreting the meaning of the results through personal reflection and use of literature; and (6) conducting strategies to validate the accuracy of the findings.

During the first step, preparing and organizing the data for analysis, the researcher transcribed the 10 interviews, deidentified the data, asked participants to member check the transcripts for accuracy, and prepared the data for further analysis. The transcripts were created using the transcription service embedded in Zoom, and the researcher reviewed each transcript and audio file to ensure the transcript was accurate and contained verbatim dialogue. The researcher also numbered each exchange within the individual transcripts to allow for easier analysis. The researcher also made sure to remove personally identifiable information and deidentified the data. A master list was created and included the participants' names, schools, years teaching, grade level, and e-mail addresses. The participants were assigned pseudonyms, each of which was connected to them on the master list, as well. The transcripts, audio files, and master list were stored on a password-protected USB drive, which was stored within a locked file cabinet only accessible by the researcher. The transcripts were sent to the participants for review within five business days. Only one participant requested any corrections. Seven participants confirmed accuracy; the remaining three did not respond within five business days, and the transcripts were assumed to be accurate. Following this confirmation, the master list and audio recordings were destroyed. The transcripts were inputted into the Qualitative Data Analysis (QDA) software being used, Atlas.ti, and coded through the program using highlighting tools and organization features.

The second step in qualitative data analysis is the initial exploration of the data through the process of coding. This included multiple read-throughs of the transcripts and note-taking through these readings. The researcher read each transcript through three times with the purpose of examining relationships, analyzing patterns, and viewing the transcripts through a sociocultural lens. Notes were made based on connections to these theories. Following the read-throughs, the researcher began to code the texts. She separated the text into multiple segments, which were labeled with both in vivo codes and deductive codes down to 62 codes. The deductive codes were based on the existing literature described in Chapter 2 and the theoretical framework for the study, the sociocultural theory (Vygotsky, 1978). The codes were made within Atlas.ti by highlighting quotations and either creating new codes or applying previous codes.

In the third step, the codes were used to identify themes within the data. The 62 codes were reduced to 11 codes groups to limit redundancy by creating descriptive categories. The Code Manager and Code Group Manager within Atlas.ti was used to achieve this. By using the Code Manager, the researcher could review both the individual codes, the code groups, and the quotations from the transcripts associated with the codes. The codes were combined into four themes from the code groups by comparing the code groups, codes, and patterns within these categories. The themes represented multiple patterns that occurred across multiple transcripts and that appeared across the codes.

Fourth, the researcher created a visual representation of the themes and developed a written narrative of the themes. This was further developed in the fifth step, wherein the researcher connected the results of the study to personal experiences and the existing literature described in the literature review. These findings and interpretations appear in the following sections and in the fifth chapter. The sixth step of the data analysis process, conducting strategies to validate the accuracy of the findings, occurred throughout the study; the researcher also will destroy all study data remaining following the conclusion of the study after three years.

Presentation of Results and Findings

This section introduces the participants of the study and presents the results through a narrative and data tables. Eleven interviews were conducted with educators of varying experience who each described discourse to reflect their perspectives facilitating it; however, only 10 interviews were viable. The participants and their experiences described in this section come from the 10 viable interviews, each of which took between 30-60 minutes. Each of the participants is provided with a pseudonym in this study to protect their confidentiality.

Participants

Eleven participants were interviewed for this study. However, one participant's change of employment made her no longer meet recruitment criteria, and the interview data was destroyed, resulting in 10 participants. These 10 participants came from seven different schools in five states, including Connecticut, Maine, New Hampshire, Rhode Island, and Vermont. Not all schools to which recruitment emails were sent are represented in this study. One participant was a kindergarten teacher, one was a first grade teacher, two were third grade teachers, five were fourth grade teachers, and one was a fifth grade teacher. The experience of these educators teaching mathematics ranges from 8 years to 25 years. The participants' pseudonyms, years of experience, and grade level are summarized in Table 1.

Table 1

Participant Summary

Participant Pseudonym	Teaching Experience (in Years)	Grade Level Taught
George	10	4^{th}
Paula	17	3^{rd}
Kylie	16	5^{th}
Gina	12	Kindergarten
Hannah	21	4^{th}
Michael	19	1^{st}
Bridget	8	4^{th}
Beth	25	4^{th}
Mark	12	3^{rd}
Sarah	20	4^{th}

George

George is a fourth-grade educator with 10 years of mathematics teaching experience. His experiences with mathematical discourse are different now, he explains, compared to when he first started teaching. Compared to past years teaching, he now says, "I think that discourse right now actually might be one of the most important things that you have." George credits the changing curriculum and the program his school has adopted as the reason for his recent focus on mathematical discourse in his classroom. He believes one of the most important things needed for teachers to become more proficient in facilitating discourse is teacher training and professional development.

Paula

Paula is a third-grade educator with 17 years of mathematics teaching experience. Her experiences with mathematical discourse differ greatly from those of the other participants. She explains that because of remote and masked learning during COVID, her third graders lack the social and language skills needed to partake in mathematical discourse. As much as she tries to facilitate discourse with and between her students, she explains that student behaviors get in the way. The strategies that she does have to facilitate discourse can be credited to professional development and her own attempts to engage students through modeling and games.

Kylie

Kylie is a fifth-grade educator with 16 years of mathematics teaching experience. As with George, the program her school has adopted helps her facilitate discourse during her mathematics class. With the program her school uses, “You can't just give the answer, you have to explain,” she says. Students now need to be able to explain the process they use to arrive at the answer, meaning that discourse is embedded within the expectations of the program. She uses turn and talk and questioning strategies to encourage discussion between students during lessons.

Gina

Gina is a kindergarten teacher with 12 years of mathematics teaching experience. She explains that because kindergarteners are so young and at a different level developmentally than other elementary students, her experience facilitating discourse is a unique one. Kindergarteners are still learning how to hold conversations, so mathematical discourse takes place as a whole group and in short discussions. However, Gina also states that there is not enough time for mathematics in kindergarten, which limits the amount of discourse that can take place.

Hannah

Hannah is a fourth-grade teacher with 21 years of mathematics teaching experience. Hannah, too, uses a curricular program that encourages students to engage in discourse. She encourages students to come up to the front of the classroom and take on the role of teacher. To do this, though, she must create a safe classroom environment where children feel they can take risks. This type of environment, she says, is a prerequisite to being able to facilitate discourse.

Michael

Michael is a first-grade teacher with 19 years of mathematics teaching experience. He began his career at his current school as a teacher of mathematics problem solving specifically, so he is well-versed in using cognitively demanding tasks to encourage discourse. His role in facilitating discourse, he explains, is that of a coach or a guide. He uses discussion and questioning to guide students towards what he wants them to notice and learn. Students, he says, are at the center of the discourse.

Bridget

Bridget is a fourth-grade teacher with 8 years of mathematics teaching experience. She believes in using open-ended questions to engage students in discourse. However, those questions, she says, should come from the curriculum. In her old curriculum, "I feel like educators kind of just have to pick their questions and prepare their lessons to support that type of work," she explained. "I think that that's where we're seeing some inconsistencies." By having a curricular program that prepares educators with questions to ask, students will all be able to engage in discourse with high-level open-ended questions.

Beth

Beth is a fourth-grade teacher with 25 years of mathematics teaching experience. She reflected upon the reform of mathematics, as well, and said that the classroom should be student-centered. There is not as great a focus on the teacher lecturing from the front of the classroom and more questioning to get to a deeper level of understanding from students, she claims. She also believes that a key piece to discourse is celebrating successes with students, as this helps build their confidence and allows them to believe they are mathematicians. The classroom environment must be conducive to discourse.

Mark

Mark is a third-grade teacher with 12 years of mathematics teaching experience. He talked about his perspective as someone who never liked mathematics as a student. “As someone who, as a child, had a tremendous amount of math anxiety, when I approach math instruction, I look at it from a computative point of view, but also from a language point of view,” he explains. The teaching of mathematical language, he claims, is critical to facilitating discourse. Like Paula, his third graders missed critical language development due to remote learning during the COVID pandemic. For that reason, he heavily focuses on language during discourse.

Sarah

Sarah is a fourth-grade teacher with 20 years of mathematics teaching experience. She claims that discourse has allowed students to believe themselves to be creators of mathematics. Students who once viewed themselves as unable to do mathematics are now excited about creating their own “Esti-Mysteries,” inspired by the work of Steve Wyborney. These tasks not only engage her students, but they also encourage mathematical language. These discussion-heavy tasks are so popular with her students that she must regulate the amount of time she can spend on them in class. She says, “It's been a really good experience for me and for them to try to get them engaged.”

Emergent Themes

The following research questions were identified for the study:

Main Research Question. What are the perceptions of public elementary K-5 teachers regarding the use of mathematical discourse as a pedagogical strategy in the classroom?

Sub-Question 1. How do public elementary teachers describe how they use different pedagogical strategies for facilitating mathematical discourse?

Sub-Question 2. How do teachers describe how students engage in mathematical discourse while using disciplinary language?

Four main themes were identified from the data collected in the study: (1) Teachers have a generally positive view of discourse as a pedagogical strategy; (2) classroom management impacts a teacher's ability to facilitate discourse; (3) teachers rely on curriculum and professional development for strategies to facilitate discourse; and (4) teachers view mathematical language as critical to student engagement in discourse. The first theme serves as the main frame. In other words, the other themes stem from the understanding that teachers positively view discourse and want to encourage it in their classrooms. Themes 2, 3, and 4 exist in concert with each other to explain what perceptions teachers have of discourse that led to such a positive view. Table 2 shows the themes and their sub-themes that emerged from the study.

Table 2

Themes

Theme	Description	Sub-Themes
1	Teachers have a generally positive view of discourse as a pedagogical strategy.	1.1 Process over product 1.2 Limitations of discourse
2	Classroom management impacts a teacher's ability to facilitate discourse.	2.1 Classroom community 2.2 Expectations for discourse 2.3 Positioning of students
3	Teachers rely on curriculum and professional development for strategies to facilitate discourse.	3.1 Curriculum and programs 3.2 Strategies to facilitate discourse 3.3 Profession development
4	Teachers view mathematical language skills as critical to student engagement in discourse.	4.1 Role of language 4.2 Supporting diverse learners

Each of the sub-themes stemmed from the codes and coding groups created during the analysis process that assist in explaining the themes. For example, the view that the process is more important than the product in mathematics is a critical piece of explaining why the teachers in the study viewed discourse positively. Further, the limitations of discourse must also be explained to solidify the participants' perceptions of mathematical discourse; without understanding the limitations, the beneficial factors cannot be fully understood. Therefore, each of the sub-themes related to the main four themes help explain how the respective theme came to be.

Theme 1: Teachers Have a Generally Positive View of Discourse as a Pedagogical Strategy

The 10 educators all had similar understandings of mathematical discourse. Each educator was able to describe mathematical discourse as the classroom community, including both the students and teachers, engaging in some sort of communication. The educators all thought that mathematical discourse involves students explaining their thinking during mathematics class. They all believed mathematical discourse involved discussion in some way and seemed to limit discourse to verbal communication, only describing written discourse in the sense of transferring what had been discussed to assessments such as exit tickets or tests.

However, in several cases, the participants' perceptions of discourse as a pedagogical strategy only recently developed, even despite the adoption of the Common Core standards, including the expectation of discourse, in 2010 (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). Kylie explained, "This has definitely, I feel like, evolved for me. When I first started teaching… I didn't really think much about it or really do it much, but now it's an everyday thing in my class." This was echoed by George, who said he would have responded differently had he been asked just 2 or 3 years ago. Paula also explained that this was something she never had training on five years ago and rarely heard mentioned. Michael, Bridget, and Beth each also agreed that mathematical discourse is a recent pedagogical strategy for them.

The participants all positively expressed their perceptions of mathematical discourse, describing it as crucial in engaging students and providing information about student understanding. Paula displayed some hesitance about using mathematical discourse and admitted it depended on the class of students. For example, her third grade class could not “handle the level of maturity that it requires to have very much meaningful mathematical discourse,” while the fifth grade class she observed frequently had greater maturity to make it meaningful. Still, she said she recognizes the benefits of discourse and loves the idea of discourse.

Michael, too, did not always believe in the benefits of discourse when first described to him. He explained that he had approached the new curriculum adopted by his school last year, which heavily emphasized discourse, with the intent to prove it wrong. Instead, he said:

> I feel like it's important that it gets out there to educators that how valuable it is, and to try and—because I am a victim of having the bias that, ‘Oh, it's not going to work for my grade level,’ and just putting those feelings to the side and saying, ‘You know, okay, give it a try and prove that the curriculum wrong.’ That was my approach. ‘Well, I'm going to show them it can't work,’ and then it does. And it's a learning experience for both ends. But just to be more open minded that it can be beneficial from kindergarten right through and having a common language throughout the grade levels is going to help students excel right through.

One of his hesitations was that discourse would never work with first-grade students. He questioned how it was possible to engage in this practice with 6-year-olds and believed that the training he received would not apply to his work in first grade. “But when I gave it a chance,” he said, “I saw the benefits from doing those talks with the students.”

The remaining participants described discourse as a critical pedagogical strategy for mathematics. One benefit of discourse, Kylie explained, was that she was able to better identify what students did and did not understand. She said,

> I think it is extremely helpful in aiding student learning and also for me engaging in doing informal assessments, just gauging where kids are at, to listen to them talking. It gives you a better understanding of the kids’ understanding, what they understand. So, yeah, I think it's a very important part of math.

Beth also echoed the idea that by using mathematical discourse, she was able to identify how well the students understood the mathematics content: “I can see who really gets it and who really struggles. Some kids are maybe close, they're there, almost there. So, it helps me with an understanding of their understanding.” She continued,

> You learn how they think by having that discourse. Sometimes a kid will really surprise you like, “Whoa! That's really interesting in a very cool way! I didn't even think about doing it like that!” You know, we have some kids who think and make you think. And that's always a great thing.

Gina agreed that by using mathematical discourse, she could figure out “where I need to fill in some gaps, or it can fill in gaps for me.” She admitted that she was sometimes surprised by what students did not know but said that discourse allowed for her to discover those gaps to then address student needs:

> I think by having it be a discourse, it engages them more so when I can promote that, then I'm promoting engagement, particularly with those that are really motivated to learn and

> don't have other issues that are distracting them from their learning. And I think that sometimes they surprise me, you know, with the things that they say, and it really adds to the instruction, and then sometimes they surprise me with their lack of understanding during the discourse, and I suddenly realize, "Oh, I thought you got that, but you did not get that." So, it can work both ways.

George also said that discourse helped him identify the needs of students. "It allows that conversation, and that discourse allows me to be able to jump in quicker," he said, referring to students who were not understanding the concept. He could then use questioning strategies to guide those struggling students to the correct path, which in turn improved their understanding. In fact, not only did discourse allow gaps to be uncovered or student understanding to be clarified, but Bridget also argued that by using mathematical discourse as a pedagogical strategy, she was able to promote student achievement. She explained, "I think that providing the opportunity for discussion, providing the opportunity for students to kind of practice their work, I think that it's helping their instructional outcome." Hannah also supported this claim by stating that mathematical discourse "strengthens their logical reasoning skills." Mark agreed that discourse improves student learning but stated that consideration of where discourse is used in a lesson is critical for building understanding. He encouraged it to be used in the second part of a lesson, after the topic had already been introduced, because "it's another layer in giving them the tools to master the concept, or at least to understand the concept."

In addition to achievement, Sarah emphasized the level of engagement that mathematical discourse promoted in her classroom. Students became so engaged and had so much to discuss during class that she had to limit the amount of time spent on specific activities:

> They got to the point where they wanted to have something to say about the pictures where I'd have to like, finally… If it wasn't an Esti-Mystery, if it was just a Which One Doesn't Belong, "Okay, now, I need to shut it down. We have one more minute where you can share your ideas but that's it." But it was—it's been a really good experience for me and for them to try to get them engaged.

Beth also found that engagement was a significant benefit to using mathematical discourse and said, "I think, with anybody who is an active participant in the process, it keeps someone engaged. Because, you know how kids can be very easily distracted by the bright, shiny thing that goes by? You keep them engaged in talking and part of the process." She continued,

> It's just really, really, vitally important that that discourse goes on throughout the entire math class on a daily basis. It's really important that you have that back and forth, student-teacher discourse, student-to-student discourse, all the time. Not only is it helpful for them to learn the math language but it keeps them all engaged, paying attention. It's really important.

Mathematical discourse also allowed students to view mathematics in a more positive light, which increased engagement, according to Sarah. Sarah said, "I've noticed, as the years gone on, some students who don't think that they're quote-unquote 'math students,' by the end of the year, they really start contributing to the discussion… I think discussion within the math classroom is an amazing tool."

To better understand how these perceptions came to be, it is necessary to examine the journey faced by the participants. Both the focus on the process over the product in mathematics and the limitations of discourse inform why discourse became a positive pedagogical strategy for the participating educators to use. The participants describe specific aspects of discourse that have positively affected their perceptions.

Process Over Product. The educators who participated in this study admitted that mathematics instruction has changed over time. One of the changes that participants identified is the idea that the process is more important now than the product. In other words, the depth of a student's understanding is more important than their ability to arrive at an answer. Teachers are looking for evidence of student understanding beyond a correct or incorrect answer. Kylie says that this change in focus on the process comes from the curriculum. "You can't just give the answer, you have to explain," she said. Students can no longer just provide an answer alone; instead, students must provide their rationale for their answers on assessments.

George is one such teacher who explained that the goals of mathematics have changed over time. He said,

> As an educator, as a teacher, I think one of the changes is we always used to teach, "Here is the answer." That's not the case anymore. It's, "What questions can I ask about this problem to get actually multiple answers?" The idea that math isn't finite, and math isn't locked in at one part; that there are many, many, many questions out there that may have multiple answers; that the process is much more important than the actual product, especially for the kids; and that the conversation around what's going on is significantly more valuable, and it's actually not the answer.

Beth also supported this thinking, indicating that there has been a change in mathematics that asks that students become deeper thinkers.

> We're moving in a way where you want the kids to be able to… You need to see how they understand it, what they're thinking is, how they came about their answer. So, we're

> moving towards more of that instead of a teacher standing in front and dictating and then practicing. So, through a line of questioning of students, you need to probe their thinking through questioning as to how they understand the concept.

Sarah dismissed the idea of right and wrong answers and spoke about her concern over student thinking, as well. While she did say that getting the correct answer is important, it is not her focus as an educator anymore as it does not provide enough evidence about how thoroughly a student understands mathematics:

> But getting away from the right and wrong answer and more about—and that's what I'm telling them. It's great if you get it right, but if I don't know how you got there, then I don't know if you just guessed, if you looked at someone else's paper and wrote it down, or if you did it all in your head, so it's… I care. But if you show me all your work, and you got the wrong answer, I might see that you just made a small computation mistake, but you really have the strategy, and that's what I want to see. So, getting there, kind of questioning so that they can show me their thinking and tell me more about it.

A common question students have when learning mathematics is where the skills apply in the real world, which Michael says can now be answered thanks to the change in focus towards process. "How does this apply to real life situations, not just doing it because I asked you to do it? And when is this going to be beneficial to you? And why? So, it's not like, 'Yes, you know how to add now.' But *why* are we learning making a 10?" He explained that by developing this understanding, students increase their motivation to learn.

The way to do this, George says, is through discourse, using rich tasks and strategies that encourage building connections:

> It's in-depth conversations, exploring a problem, having their thoughts, making that schema of making those connections and then trying to predict where that problem will go on. How to not only move through those steps, but then potentially getting to a solution to that, and then maybe connecting it to another problem or another area.

Ultimately, the transition from focusing on correct answers to focusing on developing deeper understanding has supported the use of discourse, which in turn supports the transition. A focus on process over product allows teachers to use a range of strategies to facilitate discourse. As described above by the participating educators, discourse led to greater engagement in their classrooms and increased achievement.

Limitations Surrounding Discourse. It is also important to understand the limitations surrounding discourse as described by the participants. The two major limitations include lack of time and professional development. These limitations speak to the importance of mathematical discourse according to the participants. The educators wish they had more time for mathematics and want more time invested in professional development surrounding mathematical discourse.

The first limitation surrounding discourse described by the participants was a lack of time. The educators in this study expressed frustration that they felt they did not have enough time built into their schedules for math class. As a result, they often had to cut conversations short or look for time in other parts of the day. Gina captured this idea by saying, “I feel that time crunch. And so, there are times when you would love to use discourse more or use that natural exploration to bring them where you want them to go, but you just don't trust the time element.” Rather than spend more time engaged in discourse, she said, she sometimes must move on before students are ready because of the expectation to keep going.

Hannah walked through her schedule, explaining that her math class is backed up against a transition period. According to the master schedule, her math block is 80 minutes long, which is considerably longer than the other participants. Still, she says she struggles to fit in all four components of her lessons (fluency, application problem, concept, and debrief) into one class period. She said she loses some time to transitions.

Kylie, too, expressed frustration with the lack of time. She claimed that the time allotted for the program her school uses is not enough:

> This is the challenge that I've faced with this curriculum, is that I need way more time than what the program wants me to use. So, it shows you getting through all the activities and the cool down. So, cool down is kind of like an exit slip at the end of the lesson—all in one lesson, and I can never get through all of it.

Bridget lamented the missed opportunities for rich discussions that occurred because of not having enough time for mathematical discourse:

> I think time is always a problem for teachers and for instruction, and that's part of the class I'm taking right now is—I forgot who said this… I want to say maybe Heidi Jacob Hayes. She talks a lot about instruction and curriculum, and she was saying that we really need to cut our curriculum by 30% to better our instruction because we need that time, and that really stuck out to me. I was like, yes, I think that we're trying to do too much. So, I think that we're missing so many of those opportunities to have those strong discussions, and I try my best to do it in my classroom. But I'm also limited with the time and with what we need to and what we're expected to get through.

Beth said that she wished she could have spent more time teaching students how to critique the work of others in more in-depth ways, as she believes it is a valuable skill. It is also a requirement of the Common Core standards for students to be able to critique the thinking of others (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). “It just takes time,” she said, “and I'm always under time constraints. So that's part of the problem. Not that it's an excuse, but just being realistic.”

The other major limitation surrounding mathematical discourse is professional development. George argued that there is not enough professional development out there for mathematical discourse, and it is limiting the amount of discourse happening in classrooms across the United States. He said,

> You have to try to train the teacher first. If you do not train the teacher to understand what it's going to look like, what it's going to sound like and how to grow it… Teachers for years have—and I definitely am one of it—teachers very much as much as we say, “We don't lead conversations.” We lead conversations. That's just in our nature. We're there to teach, and we're there to guide. And I think with math, we're so used to guiding through steps and guiding through trying to solve problems and trying to answer problems, that our discourse is very different from what is happening with Illustrative Math and what is happening now with kids. Because we need to train our own ears to hear the difference in conversation. But we need to train teachers’ thought process on how they're going to talk about it and how they're going to allow conversation to happen without us, and I know that sounds silly. But teaching a teacher to have a conversation without them is very important, even though we very much do it, and we know how to do it, it's just important how they do it within a math environment and how they do it whole class, and how they allow all the kids to access the same materials. That is, I think, a big task that needs to happen first before it can happen in the classroom with the kids.

Kylie, Paula, Sarah, and Bridget all explained that they had been part of professional development on their own to learn more about the use of discussion. Rather than that, George said, districts should be the ones to train teachers on how to facilitate discourse in their mathematics classrooms. They each believed discourse was valuable but needed to be taught to teachers to build consistency.

Theme 2: Classroom Management Impacts a Teacher's Ability to Facilitate Discourse

The second theme that emerged from the data focuses on classroom management and its impact on discourse. The participants described classroom management in a variety of ways to explain how they facilitated discourse in their classrooms. Important factors in facilitating discourse included the classroom community built, including students' belief that they could be vulnerable in the classroom space, the expectations created for discourse, and the positioning of students as leaders in the mathematics classroom.

Paula spoke in detail about classroom management and the difficulty she had engaging her students in discourse because of student behaviors. She said,

> I really like teaching math, and I like thinking about how the kids are learning it. But to be dead honest with you, it is so hard to get to that because I am managing behavior all day long. So, I feel like to get kids to stop misbehaving, to actually attend to what other kids are saying… So, in other words, I could have kind of a dialogue, sort of back and forth with one kid at a time, but most of the time they are not listening to each other.

She described being able to engage with a single student at a time, usually in small groups. However, even in small groups, her students would not listen to each other while she tried to hold a discussion. When one student spoke, she said, the other students would not be focused. "You're so obsessed being at my kidney table with what someone else has a different color pen that they are not listening to what anyone else said about what you noticed or wondered," she said about attempting to do a "Notice and Wonder" activity with all students in a small group. She said it was even less successful in whole group. Even when engaged in creative modeling, Paula said, "In terms of mathematical discourse for the class, it really only worked with me and the kid that were talking because no one else is there, they're just all off in La La Land."

Gina, too, said that classroom management impacted the level of discourse in her classroom. While she wanted students to engage in discourse in small groups, she limited discourse only to when students were in whole group or in small group with her because of student behavior when working in unmonitored small groups:

> But then I also find that the atmosphere of the classroom doesn't lend itself to that small group discourse because they are loud. You know that it's just the nature of having small groups of 5 and 6 year olds. But there's only one adult in that one group that it doesn't stay quiet enough. But you really feel like the discourse is meaningful. So I think that that is an area where I find it falls apart.

Thus, for students to engage in meaningful discourse, the teacher must consider classroom management as a key factor to its success.

Hannah described the need for accountability from students and explained that this was a major focus for her classroom before facilitating discourse. This is not only relevant to mathematical discourse, she explained, but to any independent or small group work happening during the school day:

> I think kids need to know why it's important that we take so much time to focus on our conversation with one another and accountability as well. I stress that at the beginning of the year. They know that word within a couple of weeks into the school year. I always

> tell them because even in our… so with in math, you know, they're without me for a period of time and, ELA, they're without me for a significant period of time as well, and because we do rotations and I tell them right away, "You know, I just need you to be accountable. I love it when you're talking, talk, talk, talk, but it has to be accountable." And we talk about what is accountable talk. We give examples: accountable, not accountable, that kind of thing, and they get it right away.

George noted that procedures and routines need to be in place before mathematical discourse can occur. He explained that these procedures have to be introduced outside of math class and, like Hannah, are generally applicable to all areas of the school day:

> I think those norms need to be built outside of that time, because I just don't think there's enough time during the actual lesson. That has to be your own classroom management because I don't think the lesson actually will allow for it.

Classroom Community. In addition to norms, procedures, and routines being developed, the participants noted that an important prerequisite to the facilitation of discourse is the development of a safe classroom community. The participants expressed the need to create an environment where students feel it is okay to be vulnerable and feel safe being wrong and making mistakes, especially since during discourse, many of these mistakes would be in front of classmates.

Sarah explained the importance she felt to foster that safe environment and said that it took time and effort to get there. In other words, high-level discourse could not begin on the first day of school because students needed time to believe in the safety of the classroom. Sarah said this was especially true because the classroom environment could impact whether a student engaged in mathematics or not:

> If you have a student that puts their work under, and they did make a mistake, and they're shaky on how they feel about it. You know, someone says, “Oh, you did this wrong,” and then you just lost that kid. So, I think both the class has to understand how brave it is for someone to come up and share their work and be respectful of that and the student has to feel safe enough to be vulnerable to that. And that takes a while. I mean, I certainly wouldn't do that day one, right? You have to build that classroom community first and then get there.

She continued,

> But I do feel like I was pretty intentional about creating as a safe math environment, because I knew I had a lot of students who were anxious about math and didn't feel like they could do it and trying to create a place where they felt comfortable making a mistake and knowing that it would be okay.

Michael, on the other hand, said that practicing engaging discourse is what helped build the classroom community. He said, “It encouraged them, made them more comfortable to share ideas. There was never a wrong answer that we didn't talk about. It was always a discussion base. So, it built a classroom community as well.”

Mistakes were especially important to the participants. Beth remembered being a student and being afraid of making mistakes: “It's hard because they don't want to be wrong. Maybe they're a little embarrassed by it, and I know how they feel, because I felt that way, too.” Recognizing that, each of the participants felt that mistakes were a method through which students learned. It was important to each educator that they create a classroom community where students felt safe enough to make mistakes. They also emphasized the ability to learn from mistakes and encouraged the use of discourse as a way of talking through mistakes and learning from them. For example, Gina spoke positively with her students when they made mistakes:

> I think I say many times through the year. I'm like, "I am so happy that happened because you know what? Now we're going to learn." So, I think that you have to really set up a culture where you embrace mistakes, too, and mistakes are not the end of the world. And I'll tell them, "Your brain just grew. Your brain just grew." I don't know if they believe it, but I definitely… I hope that it creates an environment where they're just a little more willing to try.

Hannah explained that the risk-taking mentality and the vulnerability to make mistakes began for her at the beginning of the year and was part of the development of her classroom community during the first week of school. She also felt it was important that students saw that she, as the teacher, also made mistakes. These pieces worked together to create the safe community, which in turn allowed students to build self-confidence:

> Kids have to feel safe to be risk-takers. So, we talk about that first week of school that this has to be a safe zone for everyone. Everyone needs to feel comfortable in order to encourage that risk-taking. Also noticing who needs that boost in self-confidence, so that they are willing to share, because some children have ideas, but they lack the confidence in order to share. And having children understand it's okay to share something that is wrong, or is not quite accurate, or you know, and we talk about that. And I do. I make mistakes throughout the year, too, and you know, and we'll stop and I'll say, "See, everyone makes mistakes," you know, and that's why you have to be a risk taker. You have to be willing to share.

Sarah wanted students to know it was okay for math not to be a student's favorite subject, but also wanted every student to feel safe participating despite that. She would begin the school year by having a conversation with students that encouraged them to feel safe making mistakes, which she hoped would help them feel safe to participate in discourse.

> I try to be conscious about it. And I'm trying to think of the right word. Just purposeful about it. Just to say, "Everybody," we just had an honest and open discussion, "Not everybody loves math. And I want you to be able to be okay with making a mistake, and sometimes when you make the mistake, you learn the best when you are able to figure out where you're thinking got off track."

Mark liked to compare math class to sports and encourage mistake-making by comparing it to soccer practice:

> You know, I always encourage kids, like, "Guys, truthfully, like this is, you know… there's a difference between soccer practice and a soccer game. This right now, math classes is math practice. So, this is where you make mistakes. This is where you want to ask questions and fall down so that when we have the 'game' quote unquote and we have the 'test,' you know your skills are ready and you're sharp."

Similarly, Paula preferred to use games to minimize the feeling of risk students felt they had to take.

Like Michael, George also used mathematical discourse to develop a safe classroom environment. "A lot of it happens naturally," he said about students engaging in mathematical discourse. He started with only a few students willing to risk being wrong, but over time, students no longer felt the need to classify answers as right or wrong, and participation increased:

> As they start to interact with different materials, different questions, different answers, different ideas, different conversations with the kids, they start to build a little more background knowledge. And those kids who didn't quite speak at the beginning of the year, we're able to then have a conversation as the as the year goes on. So, it's kind of one of those things where, I think you're always hoping and looking for a hundred percent participation on the on the first day. But I don't think it has to be rushed because you want those kids to build their own confidence, and to be able to have you know their own self-motivation.

Bridget, however, found the opposite happening over time in her fourth-grade classroom. Rather than students becoming more comfortable over time, she found that students became more self-conscious as the school year progressed. Developmentally, she said, fourth graders would become more concerned about what their friends thought of them, which dissuaded them from taking risks and making mistakes. Students also did not want to participate and be correct at risk of looking too smart. Bridget needed to encourage students to feel safe participating in classroom discussions throughout the school year:

> And I noticed that—I taught first grade for four years, and for the most part I found that they were not shy to ask for help. In fourth grade, especially towards the end, they're very shy to ask for help. They want to act like they don't know. They don't want to come to the back table and have their friends see them, or they don't want to pull out the resource finder. So, that's been something that I feel is very tricky, and I'm having a hard time navigating that. So, I just talk to them a lot about, "We all have strengths and weaknesses. There's things that we're going to get. There's things we're not going to get," and just

> really having those open conversations about, they need to seek out and advocate for themselves. We talk a lot about self-advocacy and advocating for themselves.

Expectations for Discourse. Not only did the teachers need to create a safe classroom environment for students to feel willing to participate in discourse, but they also all introduced expectations at the beginning of the year that informed students how to behave and interact while engaging in discourse. These expectations provided students with guidance on how to engage in discourse appropriately, but they could be applied to multiple areas of learning rather than to mathematical discourse only. Expectations were sometimes embedded in social-emotional learning (SEL) instruction or in morning meeting time. Other teachers laid out specific expectations for discussion in all subject areas.

Mark said he starts the year generally by developing social-emotional skills and then makes expectations that apply more to mathematical discourse.

> I would say, there are some baseline expectations that would probably fall more into the social emotional part of things, you know, of just how to ask questions, how to respond to somebody's comment or something like that. And as the year starts to progress and we get those soft skills under control, then we'll really kind of focus more on the discourse, on the proper use of terminology and consistent use of that terminology.

Students become so good at following expectations during discussions, Sarah said, that they self-monitor. "Everybody needs to be listening. We're gonna wait until they're quiet. And they, oh, goodness! They get so good at that. 'I'm gonna wait until everyone's quiet before I start talking.'" For Beth, students constantly use language such as, "I'd like to add to what Jessie said," which is part of their discourse expectations.

Michael also develops these skills as part of the SEL instruction, relevant to all parts of the school day, and then transfers them to mathematical discourse:

> Those expectations that we developed pretty much early on in the school year. There are, you know, norms that we set for classroom discussion, and I also tie that into my SEL portion of the day, too. The norms are consistent across the curriculum. How when someone else is speaking, we're listening, and all of those things that are helpful to make a discussion go as intended and keep on point.

Hannah worked with her students during the first week of school to figure out what expectations had to be in place to help students be successful engaging in discourse. She set basic routines with students and then had a conversation with them about what they need to be successful. She did this in a sharing circle that felt less vulnerable for students. She said that it is this time that most of the routines for communications are put in place. Bridget and George did the same, building these routines during the first couple of weeks of school. Guiding questions for George included, "How do you act? How do you do X, Y, and Z? And what does it look like? What does it sound like? What are the behaviors, what are your expectations? How do we communicate? What are our rules for each other?" He said that this was all done outside of math class because of time but that these expectations formed the basics for engaging in discourse. Kylie also developed her expectations at the beginning of the year with the guiding questions, "What are things that are appropriate, what we should be talking about, and also, if you disagree with what someone says, how you do that? How do you disagree with somebody? How to be respectful?"

Paula also developed her expectations with students at the beginning of the year during her morning circle. However, she expressed that students could not transfer these expectations from circle time to math class: “They are the concrete thinkers still, so, ‘This isn't the morning circle, so I don't have to do it there.’” She believed this was the result of not having this transfer modeled during the COVID pandemic, as third grade was the first time they were expected to do so since the kindergarten disruption.

Gina said that modeling is a key way to share expectations for discourse. When building expectations, she focused her modeling on what discourse should look like. When things do fall apart, as Gina said they are bound to do, the teacher needs to have a plan for bringing it back together. In her class, when things do not go as intended, Gina said she will “kind of talking afterwards about what that rule looks like and what's okay and what's not.”

Positioning of Students. Lastly, the positioning of students impacts the classroom management that also informs how successful teachers will be at facilitating discourse. Positioning of students can happen in a variety of ways as described in the literature (Bennett, 2014; Smith, 2021; Sullivan, 2019; Wilson & Smith, 2022; Woods, 2021). The participants described positioning of students in three ways: positioning as teacher, creator, and mathematician. Where students were positioned as teachers, they took over the role of More Knowledgeable Other (Vygotsky, 1978). Students who were positioned as creators were in charge of creating mathematical opportunities for other students. Students who were positioned as mathematicians were able to identify themselves as the do-ers of mathematics as opposed to the receptors.

George described the positioning of the student as teacher and explained the benefits both to that student and to the students being taught. He would ask one student to go to the front of the class to explain their process of solving a cognitively demanding task. The rest of the students would observe the process. This could also occur in partners or small groups. He described,

> This person is going to be the teacher, the other kids are going to be the students. “You're gonna teach them through this process. You're going to watch and learn. And then maybe tomorrow, you're going to flip those roles.” And that is, one, good for the teaching student, because now they're trying to break down their own thought process. You know, they're trying to walk themselves through it. And at the same time, those kids who need those skills are seeing it broken down in a kid friendly way. They're listening to it again, so it's a second time that they hear it. They are more confident because it's a peer that's working with them. They will be walking through step-by-step with those other kids to do it.

Michael also agreed that the other students who were not the teachers also benefitted from the positioning:

> If one student was very strong making tens or doubles plus one strategy and other students were still working toward basic manipulative solutions to problems, manipulating it with different tools, they were more encouraged to try to use the different strategies because they saw their pairs doing that.

Sarah employed a similar strategy to George in her class, where one student would come to the front of the class and display their work. She said, “Students could… show [their work] on the board or something like that, and we could all talk about it. And that really got some of them… got to where they enjoyed teaching the class how they were doing something.” It gave students a sense of pride and ownership and added to the classroom environment as a safe place for students to speak and make mistakes.

Beth would adapt this role slightly and make a student the teacher's assistant. This would remove some of the stress of being the only one in front of the class. She would provide some guidance, but the student would essentially lead the class through the work:

> I call them up to the front of the classroom and I have them help me solve it. Now, I might have to say, "What do you think is the first step?" and sometimes they can accurately guess. "See, look at this, all right! Let's work on this part together. All right. What should we show the class next? What do you think?" And they may go, "Okay…" "Let's think about this!" and kind of look at the problem.

Similarly, Bridget would take some of the stress off less confident students by having another student explain someone else's work. "If they have a hard time using the language behind it, they'll kind of write out what they did, then I'll call another kid up to try and explain what he thinks they did," she said.

Students could also be positioned as creators. Sarah allowed students to create their own Esti-Mysteries, inspired by the work of Steve Wyborney. These Esti-Mysteries provided students with clues to guess how many of a certain object is in a vase. Students in Sarah's class loved the Esti-Mysteries so much that they would create their own to share with the class. She would have dedicated time for students to share their Esti-Mysteries. This encouraged engagement in mathematics and promoted mathematical language, as the Esti-Mysteries relied on mathematics-specific language to solve.

Sarah also positioned students as creators by writing down a student's strategy as an example for the class. "How great is that to see my name up there with this strategy attached to it?" she asked, referring to the work of a student. "Like, 'Oh, yeah! That's mine!' And then having someone else say, 'Oh, I want… I like Sarah's strategy.'" Students were the creators of mathematics and strategies to solve mathematics, which could be used to encourage other students.

Lastly, students could be positioned as mathematicians. George used low-risk activities such as "Notice and Wonder," which were accessible to all students. Every student learned that they had a valuable contribution to make regardless of their academic level. "They are learning they all are mathematicians because they all have something valuable," George said. He continued,

> So, that discourse as being a mathematician is that thought process that the kids have throughout the day, the learning, the activity, and that should carry on through. So, the whole group work, their small group, and their independent work is where that mathematician conversation happens, I think.

Bridget said that positioning students as mathematicians and as owners of mathematics is a time-consuming effort. "But I also think that it requires a lot of time," she said. "It requires a lot of patience. And I think it involves—it requires the teacher to kind of take a step back and allow the students to engage in those discussions and explore." On the one hand, teachers need to have excellent classroom management, but on the other, teachers must also be willing to relinquish some of the control over to the students and allow them to be the builders of their own learning.

Theme 3: Teachers Rely on Curriculum and Professional Development for Strategies to Facilitate Discourse

The third theme that emerged from the data involved the reliance the participating educators had on their curriculum and professional development for strategies to facilitate discourse. As discussed within Theme 1, the participants explained that there has been a movement focusing on the process over the product; this thinking was evident when the participants described their curriculum and the programs used in mathematics. Additionally in Theme 1, the participants mentioned that they believe teachers need professional development to facilitate discourse. Theme 3 addresses the strategies that the participants learned from their professional development and training.

Each of the participants mentioned recently piloting or adopting a new curricular program for mathematics. Programs mentioned included Illustrative Mathematics (hereinafter: Illustrative), Bridges in Mathematics (hereinafter: Bridges), Engage NY/Eureka Math, and Zearn. For full disclosure, the researcher's school recently adopted Bridges, and she will be piloting Illustrative Mathematics for the middle school levels this school year; additionally, a previous school district with which she worked used Eureka Math and Zearn. Therefore, while the researcher is familiar with each program, she may also be biased based on her own interactions with them.

Curriculum and Programs. Each of the participants described a recent adoption of a new curricular program for their mathematics instruction. Gina used Eureka at her school, which Mark's school used before switching this past school year to Bridges. Hannah and Michael recently adopted Zearn at their schools. Bridget, Beth, George, Paula, and Kylie adopted Illustrative at their schools. Sarah was an outlier and did not identify a specific program. However, she did describe the All Learners' Network (ALN), which she relied on heavily, as "not a program." Its role for Sarah was similar to that of the other programs used by participants, so ALN will be considered a program for the intents of this section.

Each of the teachers who used Illustrative agreed that the program encouraged mathematical discourse. Kylie explains the structure of an Illustrative lesson, which expects that students engage in discourse during multiple points in the lesson. Additionally, the program provides some strategies for the teacher to guide them towards what they want the students to discover:

> This program expects them, like I said, to kind of come up with the thing themselves, and if they aren't, then I use questioning to get us there. So, like with the volume example. “What do you notice about what we came up for volume and the number of cubes in a layer? And the number of layers?” So, if they don't come up with it themselves, I have to ask some leading questions so that I'm not coming out and telling them.

George, who also used Illustrative during the last school year, praised the program for moving beyond right and wrong and encouraging students to think.

> We're using Illustrative Math right now, and one of the big things is kind of these questions that in the beginning… you know, it's this, “What do I think? What do I wonder?” And it's not necessarily a right or wrong answer anymore. It's not something that kids feel intimidated about. It's just to think about their thought process of, “Okay, if I were to look at this, what could I see? What would I do, and why it might I do it?” And those questions are very different than teaching math even two or three years ago while we didn't have the program.

The “Notice and Wonder” was a common task between many of the participants, even amongst different programs. Gina used Eureka, but she also had “Notice and Wonder” built into her program. She said one of the wonderful parts of strategies like “Notice and Wonder” was that every student could access it. Even at the kindergarten level, she saw and heard students engaging in discourse in which they were able to notice and wonder about the mathematics task because of the prompts selected by the program. She said,

> Then they give me what they notice, what they wonder. So sometimes it's a picture that's related to math. Sometimes it's a number, and just kind of see what they come up with. I will say that by this point in the year I am often just thrilled with what your higher-level kids—I mean higher level kindergarten is still higher-level kindergarten—but I'm pretty amazed and thrilled sometimes. There are times when they say things that I'm like, "Wow, yes, yes." So, they really are capable of just thinking about it, even at 5.

Because of the behaviors displayed by students, Paula said that students could not attend to the "Notice and Wonder" tasks provided by Illustrative Math when done whole class as intended. Instead, she had to modify the tasks to be done in small groups. She did agree, however, "the aim of IM was to try to get them to be mathematical thinkers and use mathematical discourse." There is also a push at her school to use the program with fidelity, so she still follows the expectations of the program, albeit in different formats to adapt to her class's needs.

Sarah's ALN also provided "Notice and Wonder" tasks, along with "Would You Rather" and "Which One Doesn't Belong?" tasks. ALN also prescribed a specific format for mathematics instruction, which followed a structure similar to the programs used by the other participants. This included a launch activity that asked students to engage in discourse and a main lesson where discourse was once again encouraged in partner talk. ALN also provides a database of tasks and questions to ask.

In another program, Hannah explains one of the key pieces of Zearn is the application problem, about which she said, "Children are engaged in problem solving every day, and that is a time that we ask for a high level of engagement and communication amongst children." Children are encouraged by the program to share their thinking and work with partners throughout the problem. The strategies used to encourage discourse came directly from the program as well:

> I mean, it's always built into our programming. I mean, there's constant questioning in our math program, constant questioning in our programming encouraging kids to question and wonder as well, you know, as a huge part of it, and making sure that questions are the meaningful questions, you know, getting to the hows and the whys, explaining your thinking.

Hannah also mentioned her hesitance to stray away from the program in any form, so most of her work facilitating discourse is directly related to the program.

Gina, however, said she did not rely strictly on the program for her strategies. “Well, we are encouraged to use Eureka Math which I do not use religiously and with fidelity. And so, I will often look at their application problem, and sometimes I'll use it exact, and sometimes I'll adapt it.” She also mentioned using strategies from other resources if she did not like the one from Eureka Math. Mark also did not rely strictly on Bridges for his strategies but instead liked to use some of the manipulatives from the program and some of the application problems from Engage NY/Eureka Math.

Having a scripted program was seen as a benefit for some. Bridget praised the use of Illustrative at her school, which had only recently been adopted. Prior to this, her school used a program developed by the district math coach. While it was good, Bridget said, she liked the idea of having a program that provided higher-level questions for teachers to ask, as this would minimize inconsistencies between teachers who did not have the mathematics knowledge to know what questions to ask students. Beth also praised the program for identifying what questions she needed to ask as the teacher.

Strategies to Facilitate Discourse. The participants described their strategies for facilitating discourse in their interviews, and each participant utilized similar strategies to the others. Sometimes these strategies came directly from the programs used. For educators that used Illustrative Mathematics, for example, they all engaged in “Notice and Wonder” activities with their students. Other times the strategies they used stemmed from the professional development that they had on mathematical discourse or best teaching practices.

One of the major strategies used by the participants that was used was questioning. Paula said she used inquiry questions as her main way of engaging her students in discourse with her. “We have to role play as a teacher, right? That you don't really understand what's going on?” she explained. “And so, I would ask them to sort of defend like why they did something instead of something else.” Students enjoyed explaining their thinking to Paula, as if she did not understand. It gave them a sense of importance.

Hannah would ask questions to encourage efficiency. She would say, “Gee, I'm wondering, you know, you drew a tape diagram with twenty boxes. Is there a different way to do that? That was more efficient?” She liked to focus on the “hows” and “whys” of her students' mathematical thinking. Kylie agreed, identifying questions like, “How did you get that?” She tried to get more information from students by asking open-ended questions:

> So typically, I'll say, “Okay, can you tell me how you got that?” Or if I'm not really understanding, “Can you tell me more?” Or, “Can you explain how or why that works?” Sometimes I'll say to them, “If you're trying to explain this to a younger child, how would you say that?”

Sarah would ask similar questions: “How did you figure that out?” or “What were you thinking when you did this?” She encouraged students to walk her through their thought process and, to challenge them, to try solving the problem in multiple ways. Beth, too, would ask them, “Well, how did you get this answer?” or “Explain to me what you were thinking when you wrote this down." She would also ask, "Can you explain to the class by going to the board and showing us how you got your answer and then explain it?" Each of the examples of questions Beth gave focused on "how" the student did their work or needed to proceed.

While there are questions embedded in the programs used by the participants, Bridget said that flexibility is important, as well, and used what she learned from her professional development to ask questions. She mentioned that her Master's program on Curriculum and Instruction has focused heavily on questioning strategies:

> And a lot of times, questioning is just kind of on the spot. I feel like that's something that's really hard. You want to prepare for it and make sure that you have good essential and open-ended questions within the lesson, but I also think a lot of it needs to be kind of natural with what's going on in the classroom in that moment.

In addition to questioning from teachers, some of the participants used questions from the students to guide their instruction. Mark explained that if one student asked a question, it was likely that other students had the same question:

> I think that's the main tool that I would use, right? So, if I just keep going back to multiplication, because that's sort of easy, you know, you've got to know your audience. So, if you've got a kid asking, "Well, what does the first number look like in an array?" You know, what was statistically, what is it that 20 of the people in the class have the same question? So, I'll use those questions almost exclusively unless we get really far kind of off the rails.

Not only should the teacher use student questions to help them, but students should be able to use their own questions to guide their own thinking. George spoke to the importance of teaching students to ask questions on their own:

> Because, like I said, when they take those tests, they don't have me to walk through those steps. They need to be asking themselves their own questions and be able to ask themselves, you know, what do I see? What do I do? What do I know? What do I think? Well, I wonder… and then be able to move through without me.

Like George, Hannah taught students to ask clarifying questions. She said, “I always encourage them to ask for clarification. And it's funny, because they will say, ‘Just for clarification…’ They'll actually say that phrase to me quite a bit.”

Another strategy educators used was asking students to agree or disagree with another student and explain why that was the case. Gina would present students with a number story identified from the program. Students would be asked to draw a picture based on the number story, and then they needed to share with a partner and then explain the picture. The other student would need to say whether they agreed or disagreed with the drawing. Even at the kindergarten level, Gina said all students were talking about their math pictures. Additionally, she noticed that students would adapt their work based on the work of their partners; for example, if a student added a number sentence to their math picture, this would encourage the other student to do the same.

Hannah encouraged students to use thumbs up/thumbs down to show whether they agreed or disagreed. Students would critique the work of others by using this strategy:

> If there's any kind of error—and most times, the children will point that out. Oh, you know, and lots of times, too, I'll say, “Okay, let's give that work a thumbs up, thumbs to the side,” and if any thumbs go to the side, I'll say, “Why is your thumb to the side?” “Oh, well, I think if I had used that strategy, I would have made sure I did this,” or, “When they added, I think they added incorrectly,” so we do that as well.

Kylie also did something similar, but she emphasized the importance of doing the agree/disagree respectfully and provided them with examples on how to express disagreement:

> We talked about like, “How do you disagree with somebody respectfully?” Yeah, so, we talk about what that looks like. How do we say that without telling the kids, “No, you're wrong, but instead saying something like, “Well, this is what I did,” or them asking the other person questions. We do that, too. Ask them, “How did you get that?

Similarly, Beth did an activity called “My Favorite No.” She would put up a problem whether part of it was right and part was wrong, and students would have to analyze it and explain their thinking. To remove some of the risk, she would sometimes show the work of a fictitious person instead of one of the students.

Another common strategy used by the participants was partner talk, often in the form of “Think, Pair, Share” or “Turn and Talk.” Hannah, for example, said she would encourage students to “use your elbow partner. Turn and talk about that. Think-pair-share. We're going to think, then we're going to pair up, and we're going to share our thoughts.” Kylie constantly asked students to work with partners throughout the entire math lesson, which was encouraged in her program:

> We do a math problem of the day, and they have some quiet think time to think about it, and then they turn and talk. They turn and talk to their group or their partner. But then, throughout the lesson, they’re talking to their partners about what they're doing, what they're working on, how they got to their answer, why they think what they think.

Sarah also used “Turn and Talk,” and said, “So, I love having students, you know, take a moment of think time, turn and talk to your neighbor. What are you noticing, and then share out to me some of your ideas?”

Michael pointed out specifically that students could use their work to help them discuss with a partner. While one student might guide a conversation, the other could show their work and engage that way. Over time, they would be able to engage more in the conversation by using their work to “guide their math talk.” This would support the conversation and support students in referring to evidence.

Participants mentioned that the programs were flexible in allowing students to use whatever strategy they would like to solve problems. In some cases, the teachers asked students to use specific strategies to solve problems (i.e., solve a multiplication problem using an array) if the program requested it, but most of the time, students had the choice in how they would approach a problem. The programs required that students explain their thinking and understand the process; how they reached the answer was less important than the process. For example, Hannah said,

> They could always choose, as well, ultimately, what strategies they want to use in order to solve a problem. So, as we're learning a certain strategy, they have to show that strategy. But then there are times where they can utilize any strategy like whatever works for you. How do you want to go about solving this problem? What makes sense to you?

On the other hand, Gina explained that was it different at the start of kindergarten. “It's not quite as open-ended when you start. You're introducing so many things they don't have a bank to draw from when they come into kindergarten.” She said that as the school year continued and students learned more strategies, they could “create a bank of strategies or manipulative that they're comfortable with.”

George said that allowing students to use different strategies was beneficial for everyone as a result of discourse. If all the students used different strategies and then talked about it, it allowed for richer discussions about connections and encouraged students to find ways of solving problems that worked for them. He attributed this to the change in mathematics over the past few years, saying,

> There is no one way to learn a problem or one way to think about math anymore. It's… we would probably like, you know, John's way, Jimmy's way, Mike's way, Melissa's way, you know, and the kids would put all their ways of thinking, because it's just a way of thinking. And then, okay, our thinking connects here. And these are similar. And these are similar. Oh, well, these are very different. Okay, why? But they still get us the same place. What do you see? And that you're not seeing? So choice is now… it's not one choice. It's you got 20 kids, it's 20 choices.

Beth, though, wanted to emphasize efficiency with students. Although students had a choice in the strategy they used, they were also encouraged to consider what the most efficient strategy was. While she showed and celebrated all students' work, she also wanted students to notice that efficiency was important:

> My biggest line is, "There's always more than one way to solve the problem, and if you get the right answer, I don't have a problem with it." But in certain circumstances the efficiency… so, we'll show, "I solved it this way, and So-and-so solved it this way." So, they all get the right answer, but being efficient is also helpful, because that shows their thinking at a… Especially when we do partial quotients, division efficiency, definitely is the best way to go about it, because they can make so many—the kids who struggle, who don't know their multiplication tables and things, they get 10 steps in and then inevitably they make subtraction mistakes and things.

Lastly, educators in this study also used students' own words as a strategy to facilitate discourse. For example, Gina, Hannah, and Kylie would repeat what students said. This not only allows for the teacher to confirm their understanding of student understanding, but it also allows students to hear their own words and solidify their understanding. "Maybe I'm repeating back what I think like, 'Let me see, I think you're saying blah, blah, blah.' And maybe they weren't. But all of a sudden, they like, click, right?" Gina said. Hannah would specifically identify what she wanted students to notice from what a student sharing said and repeat based on that: "I'll interject. I'll jump in if there's something, you know, notable. 'Oh, I love how that tape diagram was modeled with the one whole. Did everybody notice that?'" Kylie would synthesize all that she heard various students say and present it back to the class to confirm: "This is what I heard you guys saying."

Sarah also used students' words and would document their ideas on the board. The purpose of this, she said, is to identify misconceptions by writing exactly what students said. Students could then discuss and correct their misconceptions. She said,

> I listen when they're turning and talking to each other, I listen to what their conversations are, and I listen to—if I have a student showing their work, and another student sharing what they think that student did, just really listening to what are they saying about it? Are they understanding the place value piece of it, or are they saying, you know, "I just cross off the 1 and put it here," or "When I multiply by 10, I add a 0," and I'm like, "Do you?" So, I write… I will write it up on the board. So, 9 times 10 is 9 plus 0. "No, that's not!" So, trying to use what they're saying and showing literally what it means and having them correct themselves with what it is. So, it's just trying to tune in and listen to what they're saying, so that I can try to catch and fix any misconceptions they may have.

Professional Development. The participants also perceived professional development as vitally important to facilitating discourse. While they had curriculum and programs they could rely on for strategies to help them facilitate discourse, the participants also believed they needed professional development to help them become more comfortable and confident using a pedagogical approach so new to them. For example, many of the participants described learning about new strategies from professional development opportunities they took on their own, while other participants described needing it to convince them of the benefits of discourse. In general, the participants agreed that professional development was a necessary component to facilitating discourse successfully.

George was especially vocal about the need for professional development for all teachers surrounding mathematical discourse. He said,

> We need to train our own ears to hear the difference in conversation. But we need to train teachers' thought process on how they're going to talk about it and how they're going to allow conversation to happen without us, and I know that sounds silly. But teaching a teacher to have a conversation without them is very important, even though we very much do it, and we know how to do it, it's just important how they do it within a math environment and how they do it whole class, and how they allow all the kids to access the same materials. That is, I think, a big task that needs to happen first before it can happen in the classroom with the kids.

He believed that discourse was a change in pedagogy and a change in overall teaching, which meant that teachers needed training on how to approach it.

Paula agreed, stating that discussion about discourse was not present in any training even five years ago, but explained that she had to do her own professional development to learn about mathematical discourse. She described reading a book about mathematical discourse and learning about strategies from that text, and she also took a workshop from the Maine Mathematics and Science Alliance that provided her with a wide variety of strategies to facilitate discourse. Other educators, like Bridget and Kylie, explained that they took graduate classes that taught them about mathematical discourse, but once again, this was on their own time, not part of the professional development provided by their districts.

Michael, too, explained that just trying to use mathematical discourse as a pedagogical strategy for math was important to him. What he wants others to know, though, is that it worked. He wants other teachers to have what he did not: professional development. He said,

> It's important that it gets out there to educators that how valuable it is, and to try and—because I am a victim of having the bias that, "Oh, it's not going to work for my grade level," and just putting those feelings to the side and saying, "You know, okay, give it a try and prove that the curriculum wrong." That was my approach. "Well, I'm going to show them it can't work," and then it does.

Overall, the participants agreed that mathematical discourse was beneficial but that they often needed professional development so they could learn about strategies to use. Often, that professional development was not provided by the school or district in which they taught. Instead, the participants had to seek out professional development. They found professional development helpful overall in providing ideas for how to facilitate discourse.

Theme 4: Teachers View Mathematical Language Skills as Critical to Student Engagement in Discourse

The final theme identified from the data focused on the use of mathematical language in mathematical discourse. The participants each spoke about the mathematical language used by students and said that it played a key role in students' engagement in discourse. For example, Hannah said when discussing the benefits of mathematical discourse: "In order to communicate their math, [students] have to understand what they're saying… to be able to… teach them what you're thinking, you have to fully understand what you know." In other words, students who engaged successfully in discourse and used mathematical language had higher achievement in her class. Mathematical language played a significant role in mathematical discourse, which also impacted achievement.

However, not all students are able to use mathematical language successfully, but they are still expected to participate in mathematical discourse. In that case, the teachers needed to put scaffolds in place to support their diverse learners. These learners were still encouraged by their teachers to participate in discourse, but they may have had different expectations that addressed their individual needs. Both the mathematical language and the scaffolds were critical pieces to the participants while explaining their perceptions of mathematical discourse.

Role of Language. Given that mathematical discourse involves the communication of mathematics (NCTM, 2010), language plays a key role in students' ability to participate. This language development begins as early as kindergarten in a K-5 school. Gina, a kindergarten teacher, said that instruction of language starts at a very basic level when students first enter school:

> When they start, they don't know how to talk about what they did over the weekend… like they don't know how to talk it in a conversational communicative let-me-really-explain-myself way. That's not how they enter. And so, you know, part of the whole year is them learning to share their thinking on any subject, and truly I mean their narratives

> don't always make sense because they're just not used to having to relay information to people in that way.

Students in kindergarten have to learn how to have conversations before they can be expected to use mathematical language. However, Gina said by the end of the year, students were engaging in discourse and talking about mathematics. She explained that she had to start with very basic tasks and have students repeat key mathematical phrases back to her to encourage them to use it. “Using new words can feel weird!” she said, so “they just need to experience it.”

Beth also asked for students to use their mathematics language and would stop students as they spoke or wrote to remind them to use it.

> “Remember when you're explaining or writing how you know this to be true, remember to use your math language.” And if they're in the middle of explaining, and they don't use it, I'm saying, “Stop where you are. Use the math language now.” So, I do, I will stop them and have them use it. Because, of course, they defer to the easiest kind of math possible, you know?

Bridget spoke about the importance of language in mathematics, as well, even at an upper elementary grade level.

> I think a lot of math, when I was younger, I always thought was just numbers, and as I've gotten older and I've started working in schools and working with curriculum developers and working with the math coach, I realized how much language surrounds math and how important that language can be in math.

Bridget further supported the use of mathematical language by tying the language to her objectives for students and writing those objectives in student-friendly ways. She ensured that the objectives used the language, and when students discussed what they were doing that week, they could refer to the posted objectives. Further, she would have students reflect on the objectives at the end of the week and identify which they hit and why. This further encouraged them to use the vocabulary. They also identified which objective challenged them, which helped inform Bridget what students needed for additional instruction.

In his classroom, George found that by speaking, students often had an easier time writing, building a connection between the two forms of communication:

> “Why is this wrong?” So many kids before would just say, “Well, because they didn't carry over the 2.” “Wow! What do you think that they might have done?” That higher level thinking is hard for a lot of kids. But if you could have a conversation first before you're writing it down, the kids are able to access that material a little more.

Supporting Diverse Learners. Paula spoke about the COVID pandemic and the impact it had on her students. She described how her students fell behind developmentally with language, leading her to make adaptations to her mathematics to account for her struggling students. Paula liked to use a strategy called “Numberless Word Problems” with her students. It took away the pressure or focus of numbers and allowed students to focus on the language being used in word problems.

> Instead of, “I noticed that there are some dogs at the Dog Show, and now there are more dogs, so I think this is the joining.” Probably they could do that. And I don't know how I found it, but I sleuthed around on the Internet, and I came across this model called numberless word problems where you could sort of slow-reveal different parts of a word problem. And it really would have things like, there are some dolphins in the water, more dolphins are coming, and then you just sort of stop and I have discussion with the class in

> that sense. “So, what do you think we're going to do? Are we going to be adding or subtracting?”

This structure supported the students in focusing specifically on the language rather than focusing on the numbers. She said often students would see the numbers and immediately solve rather than look at the context of the problem. By using a numberless word problem, she was able to maintain the context of the problem and encourage students to focus on the language instead of numbers.

Mark also noticed his school had a deficiency in language, like Paula. He said this was school-wide and teachers had to prioritize mathematical vocabulary and language,

> I look at it from a computative point of view, but also from a language point of view. And I think in my school district, we are seeing a collective deficiency in vocabulary across the board, whether it be mathematics or reading, that we have loss of skills. But vocabulary seems to be this huge hole. So, I think it's important to teach mathematical vocabulary and language during a math lesson. So, you know, if you're talking about multiplication, you're explaining and showing what, you know, factors and products are and aren't.

As such, like Paula, Mark needed to put scaffolds in place to support his students. He explained that he needed to teach these students the mathematical structure of questions asked, such as in word problems. He focused on language skills of engaging in conversation first, but once this was in place, he could focus more on the structure of word problems and how to approach them. He said consistent vocabulary was key to this.

An easy way to provide scaffolding for students was posting vocabulary where students could access it. Hannah did this and said,

> Another thing I do, too, is I post math vocabulary, so that can also be kind of a little bit of an anchor, right? If a child wants to say something, they're not sure what to say, I've noticed that they will look, and I make sure it's posted right where I teach the small group lesson, and they do. They will look up, and they will try to grab on to a word so that they have an idea so that they have something to share. So that's definitely helpful.

She also provided them with a math folder with resources students could access if they needed additional help. She said this was especially important during geometry, when all the names of shapes and the language surrounding them were new or unfamiliar. Sarah, Bridget, Mark, and Michael also utilized a word wall that students could access during discourse. Kylie and Paula identified specific words they wanted students to use and would point them out specifically, posting them or putting them on the Wheel of Math Words respectively.

Beth had a special educator in her classroom who she could rely on to support students with mathematics goals on Individualized Education Plans (IEPs). However, she said constantly reiterating the mathematics language was necessary not only for these students with the special educator, but for all students during whole group instruction:

> Not only the kids with the IEPs, but the other… we have a few of the kids who struggle—she would pull them in the back, and we just keep reiterating that math language with them. I think the more you hear something, the more it is explained how that math language is so important. Even when I have them go up to solve a problem, they know the problem, I say, “Okay, remember your math language.” We just keep, I don't want to say pounding it into them, but you just keep repeating it and repeating it and, “Nope, math language. Math language.” And they get it, it just takes some practice. They just need to hear from us all the time so they know that that's what they use.

Additional scaffolds used by the participants included encouraging students to use manipulatives to help them explain answers or encouraging students to focus on listening. Michael spoke about manipulatives and said, “So, it's very fluid in in the sense that if I do see kids that are struggling, one of those groups is at a slower pace. It's more manipulative-based, and it is just more concrete for them to see.” Mark said he needed to make sure students used manipulatives correctly but that they could be a beneficial tool to struggling students. George said that struggling students could also participate in discourse simply by listening. “Listening is really important for those kids,” he explained. “If you are a couple of grade levels behind, listening is really important, because that's the only way—listening, looking, seeing.”

Sometimes, Sarah found it helpful for struggling students to provide them with the answer:

> Yeah, sometimes what I'll do with the problem is, I will just put the answer up there, so they don't even have to… That's the right answer. That is the answer. How might I have gotten there? And sometimes those students that struggle with it might see, “Oh, you must have added, you know, 3 plus 2 to get 5, so that's the answer.” So, sometimes, putting the answer up there takes that pressure off of having to figure it out.

By taking away the focus on the answer, Sarah said, students could identify what needed to happen to get to that answer.

Lastly, students would benefit from sentence stems or sentence starters. Hannah provided struggling students with sentence starters, stating, “And sometimes with problem solving, I'll even give them stems… they always have to produce a sentence in the end, stating their answer, and sometimes I'll provide them with a stem because they get confused with the question.” Like providing the answer as Sarah did, by providing a sentence stem, Hannah said, this can take some pressure off for struggling learners. These can be used when writing about mathematics, but Hannah also provided sentence stems, or conversation cues, for verbal discourse, as well.

Kylie and Bridget also would provide sentence stems to their students. Kylie said, “Sometimes I give them, if it's a hard topic and I know they're gonna have a hard time talking about it, I sometimes give them frames, almost like I give them a sentence starter.” Bridget also used sentence stems, but she found students were even more successful when they were able to record their thinking:

> I try and put sentence starters up on the board or some prompts to help encourage students to join into those discussions. I also, since I don't have the opportunity to hear everybody, sometimes I have them record their thinking. So, they'll go out in the hall, and they'll explain to me what their work was and what their thinking was so I can listen to that afterwards. It's nice because I get to kind of get inside the kid's mind, but what I don't like about it is, I feel like I don't give immediate feedback, because it's something that I take home after. I guess I kind of think about it as, if I can listen to 3 or 4 kids explain their answer in class and 3 and 4 kids explain their answer at home, I'm hitting more than I could if I didn't have that technology available. But I feel like I'm not able to then give that that immediate feedback which is so important.

The ability to record allowed for all students to be heard and allowed for rehearsal, which would help encourage students to partake in discourse. However, as Bridget mentioned, the feedback time was a drawback to using recordings. That said, it could be a scaffold that provided support to some specific students who needed this modification.

Summary

The purpose of this study was to explore the perceptions of public elementary K-5 educators regarding use of mathematical discourse as a pedagogical strategy, where mathematical discourse is defined by NCTM (2010) as "the mathematical communication that occurs in a classroom" (p. 1). Specifically, this study was designed to explore the perceptions of elementary educators who, at the time of participation, were age 18 or older; were currently employed at a public elementary school; had at least one year teaching mathematics at their current grade level to ensure some familiarity with the curriculum; and taught in grades K-5. Individual semi-structured interviews were conducted with ten participants who met the above criteria from schools in Connecticut, Maine, New Hampshire, Rhode Island, and Vermont.

Overall, the results of the study demonstrated four themes related to the purpose of this study: (1) Teachers have a generally positive view of discourse as a pedagogical strategy; (2) classroom management impacts a teacher's ability to facilitate discourse; (3) teachers rely on curriculum and professional development for strategies to facilitate discourse; and (4) teachers view mathematical language as critical to student engagement in discourse. First, the participants spoke positively about mathematical discourse, but these perceptions evolved because of changes in priorities within the field of mathematics education, particularly the view that process is more valuable than the product. Additionally, challenges with time allocation for mathematics indicated that participants valued mathematical discourse and wanted more time to engage in it with students. Second, the classroom management of each individual teacher impacted their ability to facilitate discourse. The classroom community created at the beginning of the year, as well as the expectations surrounding discourse played important roles for participants. The positioning of students as the teachers of mathematics, the creators of mathematics, and mathematicians also allowed participants to better facilitate mathematical discourse. Third, if the participants used a curricular program that heavily involved discourse, discourse was the major instructional strategy used. Other strategies used often came from various programs or from professional development, and there were common strategies between participants. Lastly, the participants indicated that the mathematical language used by students impacted the level of discourse. The participants encouraged their students to use mathematical language when engaging in discourse but also made accommodations and introduced scaffolds for students when necessary for engagement in discourse.

The problem studied was the lack of proper utilization of mathematical discourse in public elementary settings (Fuson & Leinwand, 2023; Stiles, 2016). Results of this study indicate that the educators are attempting to facilitate discourse and believe in the benefits of discourse; however, there are many factors at play that impact teachers' ability to facilitate it. The participants also indicated that their use of discourse as an instructional strategy was recent based on changes in curriculum and professional development. The participants all saw benefits in their students' mathematical achievement when they engaged in discourse. In the next chapter, the researcher explains the importance of these findings and provides recommendations for further study.

CHAPTER 5: CONCLUSION

Since the adoption of the Common Core State Standards (CCSS) in 2010 and the subsequent *Principles to Actions* reform suggestions from the National Council of Teachers of Mathematics (NCTM) in 2014, more research has been conducted about mathematical discourse. Several studies indicate that the use of mathematical discourse in the classroom leads to increased proficiency (Anderson, 2021; Bertolone-Smith & Gillette-Koyen, 2019; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021). NCTM (2014) describes eight mathematics teaching practices in *Principles to Actions* that all educators should promote in mathematics instruction, which includes mathematical discourse. Vygotsky's (1978) sociocultural theory also states that children learn through social interactions with a more knowledgeable other (MKO). With all this in mind, educators in the United States should be engaged in mathematical discourse with students and encouraging it between students. Yet mathematical proficiency in the United States has fallen (NCES, 2022a; OECD, 2019).

The purpose of this study was to explore the perceptions of public elementary K-5 educators regarding use of mathematical discourse as a pedagogical strategy, where mathematical discourse is defined by NCTM (2010) as "the mathematical communication that occurs in a classroom" (p. 1). The researcher conducted 10 viable interviews with elementary mathematics educators representing grades K-5 in Connecticut, Maine, New Hampshire, Rhode Island, and Vermont. Criteria for inclusion in the study required that participants (a) were age 18 or older; (b) were currently employed at a public elementary school; (c) had at least one year teaching mathematics at their current grade level to ensure some familiarity with the curriculum; and (d) taught in grades K-5. Eleven educators volunteered to participate in the study and 11 interviews were conducted; however, due to a change in employment status for one of the participants, only 10 interviews were considered viable.

The 10 viable interviews, which utilized a semi-structured interview protocol viewable in Appendix D, provided information regarding the participants' perceptions of mathematical discourse related to the main research question and two sub-questions:

Main Research Question: What are the perceptions of public elementary K-5 teachers regarding the use of mathematical discourse as a pedagogical strategy in the classroom?

Sub-Question 1. How do public elementary teachers describe how they use different pedagogical strategies for facilitating mathematical discourse?

Sub-Question 2. How do teachers describe how students engage in mathematical discourse while using disciplinary language?

The participants were provided with pseudonyms, with their identifiable information stored in a password-protected USB on a master list accessible only by the researcher. Following the transcription checks, during which only one participant requested changes, the master list was destroyed. All participant information was completely de-identified.

The 10 interviews were transcribed verbatim using embedded Zoom transcription features and coded using Atlas.ti. Sixty-two in vivo and deductive codes were created from the transcripts, which were combined into 11 code groups, resulting in the identification of four major themes. The themes were (1) teachers have a generally positive view of discourse as a pedagogical strategy; (2) classroom management impacts a teacher's ability to facilitate discourse; (3) teachers rely on curriculum and professional development for strategies to facilitate discourse; and (4) teachers view mathematical language as critical to student engagement in discourse. The sub-themes of Theme 1 were (1.1) process over product and (1.2) limitations of discourse; the sub-themes of Theme 2 were (2.1) classroom community, (2.2) expectations for discourse, and (2.3) positioning of students; the sub-themes of Theme 3 were (3.1) curriculum and programs, (3.2) strategies to facilitate discourse, and (3.3) professional development; lastly, the sub-themes of Theme 4 were (4.1) role of language and (4.2) supporting diverse learners.

Interpretation and Importance of Findings

In this study, the researcher explored the lived experiences of elementary educators in K-5 schools regarding the use of mathematical discourse as a pedagogical strategy, resulting in 10 viable transcripts. Following the review and analysis of the data, four themes emerged from the data. These themes answered the main research question and two sub-questions for the study.

Main Research Question

The main research question for the study was: What are the perceptions of public elementary K-5 teachers regarding the use of mathematical discourse as a pedagogical strategy in the classroom? Theme 1, which found that teachers have a generally positive view of discourse as a pedagogical strategy, directly answered this main research question. While existing research indicates that mathematical discourse increases mathematical proficiency (Anderson, 2021; Bertolone-Smith & Gillette-Koyen, 2019; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021), there is limited research addressing educators' perceptions of their use of mathematical discourse as a pedagogical strategy. This research sought to address this lack of information and address the problem that there is a lack of proper utilization of mathematical discourse in public elementary settings (Fuson & Leinwand, 2023; Stiles, 2016).

Overall, all 10 participants spoke positively about mathematical discourse, particularly because of the emphasis on the process students took rather than the focus on the answer, which many of the more experienced educators said was a recent development in their pedagogy. Although the Common Core State Standards (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010) were adopted in 2010 by the states where the participants worked, the educators in this study described the change in pedagogy being even more recent: within the last 5 to 10 years. The National Council of Teachers of Mathematics released their *Principles to Actions* in 2014, in which mathematical teaching practices were described. Mathematical discourse appears as Mathematics Teaching Practice #4.

These seemingly recent changes described by the participants are at least partially the result of changes in curricular programs. Many of the participants described recently piloting or adopting new programs. The programs used included Illustrative Mathematics K-5 (hereinafter: Illustrative), Bridges, Engage NY/Eureka Math, and Zearn. These programs each meet expectations based on criteria from EdReports (2023b), which reviews curricular programs for alignment to CCSS and usability (EdReports, 2023c). This means that each program is aligned to standards, including the mathematical practices that require the use of discourse. Therefore, the participants each recently began using mathematical curricular programs that required them to facilitate discourse, which may not have been the case prior to the adoption of these programs if prior programs did not meet standards.

Further, the participants generally agreed that when they facilitated mathematical discourse, they were able to gain a better understanding of student progress and proficiency. Mathematical proficiency is measured by five interwoven strands: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition (NRC, 2001). Groves (2012) explained that students need to be able to understand and represent mathematical concepts to demonstrate conceptual understanding. Collins (2011) described procedural fluency as the ability to use procedures appropriately and flexibly. Groves (2012) and Collins (2011) explained strategic competence as mathematical problem solving. Adaptive reasoning requires that students adjust their thinking with novel tasks (Muin et al., 2018). Lastly, a productive disposition means that students believe themselves to be mathematicians (Collins, 2011).

The participants explained that they were able to identify student proficiency, especially adaptive reasoning, by engaging in discourse. For example, participants explained that engaging in discourse allowed students to develop their critical thinking skills by giving them additional strategies with which they can approach new tasks. Discourse, as facilitated by the educators, also increased strategic competence, as students were able to connect the concepts learned in one lesson to concepts in a new one, typically by referring in discussions to their prior schema.

The second theme, classroom management impacts a teacher's ability to facilitate discourse, further addresses this main research question. This theme's three sub-themes, classroom community, expectations for discourse, and positioning of students, explained how the participants could or could not facilitate discourse. The participants described the need to create a safe classroom environment for students to feel comfortable engaging in discourse. To facilitate discourse, educators first needed to create a classroom environment where students felt safe taking risks and being wrong. This helped contribute to students' productive dispositions, or the students' belief that mathematics could be useful in everyday life and one of the strands of mathematical proficiency (Groves, 2012). The participants in the study found that students who lacked a productive disposition began to engage more by the end of the school year after engaging in discourse. By facilitating discourse, the participants were able to help students find usefulness in mathematics and believe they were capable of using mathematics. Participants claimed that discourse was a main reason why this happened, as students felt they had something they could contribute to every conversation and understood that their contributions were valuable.

Overall, the participants found that by focusing more on developing deep understanding in their students through discourse, students became more mathematically proficient. Students learned the how and the why of concepts, they said. The reverse felt true to many of the participants, too. By focusing more on the process of mathematics rather than the product, the educators felt they were able to facilitate discourse more easily, which also allowed them to perceive discourse as positive. However, while this study revealed that teachers generally viewed discourse favorably, this favorable view was impacted by their own classroom management, the curriculum they utilized, and the mathematical language skills of the students. Thus, the sub-questions help explain the overall understanding of the main research question.

Sub-Research Question 1

The first sub-question for the study was: How do public elementary teachers describe how they use different pedagogical strategies for facilitating mathematical discourse? Theme 2, classroom management impacts a teacher's ability to facilitate discourse, and theme 3, teachers rely on curriculum and professional development for strategies to facilitate discourse, both address this sub-question.

Strategies for Facilitating Discourse

The existing literature regarding mathematical discourse describes specific strategies known to help teachers facilitate discourse (Anderson, 2021; Bennett, 2014; Bertolone-Smith &

Gillette-Koyen, 2019; Garcia et al., 2021; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Silva, 2021; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021; Wilson & Smith, 2022; Woods, 2021). Martin et al. (2015) explained that the strategies that teachers used to facilitate discourse impacted the discourse that occurred in the classroom. The most useful strategies for facilitating discourse, as described in the literature, include questioning, critiquing of student work, establishing clear expectations, thoughtful positioning, scaffolding, and the selection of cognitively demanding tasks. Each of these strategies were utilized in some way to help the participants facilitate discourse in their classrooms.

Theme 2 regarding classroom management identified the strategies of clear expectations and thoughtful positioning in the participants' classrooms. Bennett (2014), Ghousseini et al. (2021), Smith (2021), and Smith et al. (2020) each described clear expectations as a precursor to discourse. Without expectations and a culture of participation in place in the classroom, students would be unable to feel safe enough to engage in discourse (Bennett, 2014). Both the sub-themes of classroom community and expectations for discourse relate closely to these claims from the literature. The participants in the present study found that creating a safe classroom community was a prerequisite to the facilitation of discourse; that is, the participants needed to create a classroom culture where students felt comfortable being wrong or else they would not risk engaging in discourse. Part of this involved celebrating mistake-making, but the participants felt that being clear with their expectations and their development of a classroom community at the very start of the year was key. The participants explained that they start off the school year building their classroom communities through social-emotional learning structures that set the foundation for mathematical discourse.

Further, the participants also introduced clear expectations at the beginning of the school year for what discourse, whether in math class or in other subjects, would look like and sound like. For many of the participants, this occurred during sharing circles outside of math class and led to the establishment of general classroom norms and expectations for participation. While the literature described identifying mathematical discourse-specific expectations (Ghousseini et al., 2021), none of the participating teachers described doing this. Rather, they used general expectations that could apply to any class period.

The existing literature also described thoughtful positioning as a pedagogical practice to facilitate discourse, and this was present in the participants' descriptions of their facilitation of discourse, as well. Bennett (2014), Smith (2021), Sullivan (2019), Wilson and Smith (2022), and Woods (2021) each explained that the mathematics classroom needed to be student-centered, and students had to be given mathematical autonomy and authority. The participants described three ways of positioning students: as teacher, creator, and mathematician. The participants would ask their students to either individually teach other students who needed assistance or encouraged them to teach the entire class about the strategy used. Other participants asked students to create problems to share with the class or used work from a student to serve as an anchor chart. The participants all felt it was important to position the students as mathematicians and described the change from the beginning of the year to the end of the year, where students who once feared mathematics began to view themselves as mathematicians. This type of positioning in previous studies led to an increase in the amount of mathematical discourse present in classrooms and an increase in correct responses (Smith, 2021; Sullivan, 2019; Woods, 2021), and the participants in the present study perceive this as the case, as well.

By positioning students as teachers, creators, and mathematicians and providing them with mathematical autonomy and authority, the educators also set the students up as more knowledgeable others (MKOs) for their peers. The concept of MKO was described by Vygotsky (1978) in his sociocultural theory. Sociocultural theory states that when a child works collaboratively with a MKO through repeated interactions, the child internalizes strategies described by the MKO over time (Vygotsky, 1978). This is true with mathematics, where students can develop deeper mathematical understanding by consistently working with a MKO (Deogratias, 2022; Steele, 2001). When the participants in the present study positioned students as MKOs for other students, they helped create environments where children could learn through these repeated interactions necessary for learning.

Other strategies described in the existing literature included questioning, critiquing of student work, and the selection of cognitively demanding tasks. Questioning was one of the most significant strategies identified in the existing literature surrounding mathematical discourse to the point that in a study by Bertolone-Smith and Gillette-Koyen (2019), all but one interaction between the teacher and her students was in the form of a question. In the present study, participants all described utilizing questions to facilitate discourse, engage students, and clarify understanding. Questions often focused on elaboration with the purpose of getting to deeper levels of understanding. Rather than ask about answers, the participants said they tended to ask "how" and "why" questions that allowed for greater student contribution to discussion. They also felt that using these types of questions helped them in guiding students towards whatever the learning goal was. However, most of the questions asked served the purpose of helping students dig deeper into the mathematics.

Smith et al. (2020) identified two types of questions that teachers asked, including assessing questions and advancing questions. While the assessing questions sought to help teachers identify student understanding, the advancing questions guided students towards learning goals. The participants in the present study tended to utilize more assessing questions based on their perceptions of the questions they asked. They described trying to get more information from their students or encouraging students to explain their thought processes. Questions such as, "How did you get this answer?" or "Can you explain what you were thinking when you did this?" were perceived by participants as providing a greater picture of student understanding. The participants did not indicate that they utilized advancing questions, though many of the participants indicated they decided what questions to ask based on those embedded in their curricular programs. If their program encouraged advancing questions, the participants likely used advancing questions, even if they did not perceive they did, especially since they noted trying to help students progress towards the learning goals.

Critiquing student work, based on Mathematical Practice 3 (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010), was described by participants as a major challenge at the elementary level. While this practice is one that is expected to be used in all grade levels K-12 (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010), the participants explained that the language skills of younger students were limited, resulting in the need for scaffolds such as sentence stems and word walls. However, Varhol et al. (2021) described eight interactions leading to critiquing of student work: getting in contact, locating, identifying, advocating, thinking aloud, reformulating, challenging, and evaluating. Advocating and reformulating especially allow students to critique the work of others. Advocating is a process in which students examine the ideas of themselves or their peers, while reformulating is a process in which students restate the ideas of others either in their own words or with additional commentary. The participants in the present study spoke about the critiquing of student work with hesitance, with every participant explaining that they felt this was the most difficult thing to do with students in elementary schools. This was especially true with the participants who taught kindergarten and first grade, who said their students could not do this until close to the end of the year after significant pre-teaching and modeling, but even the participants who taught the upper grades described it as a challenge. However, overall, what participants described about their work facilitating the critiquing of others' reasoning fit into the two categories of advocating and reformulating described by Varhol et al. (2021). For example, many of the participants described asking their students to agree or disagree with their peers, and when they disagreed, they needed to explain why. Participants also described asking their students to restate what their peers said and to add on. For example, one educator taught her students to say, "Just for clarification," and restate what they heard in their own words. Thus, even though participants described this particular mathematical practice as the most challenging for students, the work they did to facilitate the critiquing of reasoning of others aligned with the research of Varhol et al. (2021).

Lastly, research indicates that the selection of cognitively demanding tasks also affects teachers' ability to facilitate discourse (Martin et al., 2015; Smith et al., 2020; Wilkerson, 2022; Wilson & Smith, 2022). Smith et al. (2020) argued that the use of cognitively demanding tasks was what made discourse possible, while Martin et al. (2015) found that students could more effectively engage in problem solving and in discourse when cognitively demanding tasks were utilized. The participants in the present study did not describe the tasks they used as cognitively demanding, but they indicated that the tasks used were "low floor, high ceiling" tasks with multiple entry points for diverse learners. These "low floor, high ceiling" tasks described were perceived by the participants as strongly aiding in the facilitation of discourse and supporting rich discussion. However, they almost exclusively pulled tasks from their curricular programs. This suggests that the use of cognitively demanding tasks depends on the curriculum rather than the teacher, especially since many of the participants described their reluctance to stray from the program.

Curricular Programs

As described above, the curricular programs used by the participants were perceived as playing a key role in facilitating discourse. The programs utilized by the participants, which included Illustrative, Bridges, Engage NY/Eureka Math, and Zearn, each meet the expectations of EdReports (2023b), meaning that the programs are all tightly aligned to standards. Tight alignment to standards requires use of mathematical discourse as described in the CCSS (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). For this reason, to answer this sub-research question explaining how teachers describe the strategies used to facilitate discourse, it is necessary to understand the programs themselves to explain how the participants in this study facilitated discourse while using these programs.

In its design principles, Illustrative, which over half of the participating educators used as their primary curricular program, cites Vygotsky's (1978) sociocultural theory, stating that community is a critical feature of learning (Kendall Hunt, 2021). Illustrative also specifically describes three of the five strands of mathematical proficiency: conceptual understanding, procedural fluency, and adaptive reasoning (Kendall Hunt, 2021). The publisher explains that the use of these strands contributes to the rigor of the problems utilized, or the cognitively demanding tasks used. Additionally, the design principles also note the importance of community in learning mathematics and describe several of the strategies that research has indicated support mathematical discourse. For example, they claim that the program supports "structures that establish a mathematical community, establish norms, and invite students into the mathematics with accessible content" (Kendall Hunt, 2021, para. 15). Establishing norms and building a community is supported by existing research for facilitating discourse (Bennett, 2014; Ghousseini et al., 2021; Smith, 2021; Smith et al., 2020). Instructional routines used in Illustrative units are further described as purposefully promoting discourse (Kendall Hunt, 2021). The design principles also specifically encourage teachers to be conscious of how they position students and describe the role of the teacher as one that supports and encourages rather than taking a teacher-centered approach. This is also supported by existing literature as helping to facilitate discourse (Bennett, 2014; Smith, 2021; Sullivan, 2019; Wilson & Smith, 2022; Woods, 2021). The design principles described by Kendall Hunt (2021) as integral to the Illustrative K-5 program suggest that if educators use the program with fidelity, they will be able to facilitate discourse.

Another program utilized by the study participants, Bridges, does not provide as clear design principles as Illustrative; however, a study determining the effectiveness of the program found that teachers who used Bridges reported that it was interactive, collaborative, and promoted critical thinking (SEG Measurement, 2018). A document reflecting the beliefs of Bridges reports that the company believes that learning mathematics is a social process and that teachers must facilitate discourse (The Math Learning Center, n.d.). Within this document, the company also explains that they believe teachers should utilize questioning strategies to help students learn and engage in mathematics. They describe students as actively engaging in the mathematics learning process through discussion.

However, unlike Illustrative's design principles (Kendall Hunt, 2021), Bridges does not explain how they expect teachers to use questioning or describe how students engage in discourse (The Math Learning Center, n.d.), though it may be clearer in teaching documents. EdReports (2016) also suggests that Bridges does not fully adhere to the mathematical practices (NCTM, 2014) and writes, "Additional teacher assistance is needed in engaging students in constructing viable arguments and analyzing the arguments of others" (para. 1). If the participants relied on Bridges alone as their way of facilitating discourse, it is possible that it would not be enough to promote rich discourse. While it is apparent that Bridges aims to help teachers facilitate discourse, educators might need to supplement it with additional strategies to facilitate higher levels of discourse with students.

A third program utilized by participants was Engage NY or Eureka Math. Great Minds (2023) describe Engage NY and Eureka Math as the same curriculum, but Eureka Math provides additional support resources for parents, teachers, and students. For this study, the researcher chose to focus on research about Eureka Math rather than Engage NY. A blog post discussing the design principles for Eureka Math explains that one of the one of the conditions that must be met for students to learn involves collaboration (Taylor, 2022). Two of the instructional routines described in Eureka Math, Math Chat and Always Sometimes Never, are described as promoting discussion and are used to encourage movement of mathematical skills to long-term memory. Beyond these two instructional routines, however, mathematical discourse is not discussed. EdReports (2023a) indicates that Eureka Math fully meets standards, receiving full marks for alignment. This suggests that Eureka Math does support the mathematical practices, including use of discourse. Eureka Math, in fact, tied with Illustrative for having the highest Focus & Coherence and Rigor & Mathematical Practices scores of the programs used by the participants (2023b). Illustrative and Eureka Math were the only two with perfect scores in these areas, indicating the strongest ties to the mathematical practices, including facilitation of mathematical discourse.

Lastly, some of the participants in the present study also utilized Zearn, a program derived from but separate from Eureka Math (Zearn, 2021). One of the learning principles described is, “Flexible learning environments deepen engagement and understanding,” (p. 6), which the authors claim include student engaging in dialogue about the mathematics being learned. Zearn (2021) describes the change from focusing on answers to a focus on the process and notes the connection between this change and discourse. Further, they explain that the role of the teacher is to facilitate this discourse by creating a safe environment for students to participate, which participants in the present study claimed was a prerequisite to students engaging in discourse. While Zearn (2021) does not explain in their learning principles how teachers facilitate discourse, they do cite the benefits of discourse, noting that discourse promotes critical thinking and problem solving. As with Bridges, EdReports (2021) found that Zearn did not fully adhere to the mathematical practices, suggesting that Zearn alone is not enough for facilitating discourse, and educators need other strategies to support the practice.

Given that the educators in this study perceived the curricular programs as integral to facilitating discourse, it is vital that the programs used are tightly aligned to standards and the mathematical practices. Because EdReports (2023b) suggests there is tight alignment even though some of the programs do not fully address the mathematical practices, the programs as a whole do help educators facilitate discourse but may require supplemental strategies to promote rich discourse. Participants in the study described learning about additional ideas for strategies from professional development opportunities. Thus, while the curricular programs must provide the instructional routines necessary for facilitating discourse, educators may need support through professional development to engage students in rich dialogue.

Sub-Research Question 2

The second sub-question for the study was: How do teachers describe how students engage in mathematical discourse while using disciplinary language? The fourth theme, teachers view mathematical language skills as critical to student engagement in discourse, helps answer this question. Sociocultural theory (Vygotsky, 1978) also provides the context to understanding how the theme relates to the research question. Sociocultural theory describes how social interaction supports learning in students and has three main parts: the importance of social interaction, the use of language in learning, and the zone of proximal development (ZPD). According to Vygotsky (1978), social interactions are necessary for students to learn. Children engage in interactions with more knowledgeable others (MKOs), and by working collaboratively with the MKOs, the child internalizes strategies through repeated practice. Students eventually internalize language used in these interactions as private speech, which allows for them to be more successful with complex tasks like cognitively demanding tasks. Lastly, the ZPD describes an area between two points (too easy and too hard) where students learn with scaffolds. This theory provides a foundation for understanding the role language plays as described by the educators in this study.

Beginning at the lowest grade level, the kindergarten-level participant described language as very basic but critical for her 5- and 6-year-olds. This participant explained that students at this grade level spent the beginning of the year learning how to engage in. As the year progressed, students could extend their conversations to mathematics. This required significant scaffolding, she said. For most kindergarteners, their ZPD was using mathematical language in general, with students needing supports in place to allow them to engage in mathematical discourse. The participant explained that this mostly revolved around repeating key mathematical phrases and using basic tasks that allowed for easy access to math.

Scaffolding was not present only at the kindergarten level, however. All the participants described using scaffolds to support their students with diverse needs. The expectations from the participants were that all students eventually needed to participate in discourse by the end of the school year, and with students who all had different ZPDs, the participants needed a variety of scaffolds in place for all students to be successful. Because many of the participants described seeing a deficiency in language skills following the COVID-19 pandemic, the scaffolds were viewed as especially important to them. Common scaffolds used by the participants included sentence stems, word walls, resource folders, use of manipulatives, discourse rehearsal, and even providing the answers in advance to some students. The participants claimed that since the focus had shifted from the answer to the process, providing students with the answer in advance took away some of the pressure and allowed students to focus more on explaining how they could reach that point. The scaffolds used varied by student, the participants, said, which accurately reflects the expectations of the sociocultural theory (Vygotsky, 1978), as every student has a unique ZPD with unique scaffolding requirements.

The participants at every grade level also described viewing language as key to students being able to engage in mathematical discourse. They believed that discourse extended not only to conversation but to students' written responses, as well, and explained that when students communicated verbally about mathematics, they had an easier time writing down their explanations after. This is supported by Vygotsky's (1978) and Allman's (2022) claims that students need to discuss their processes to turn the language into private speech and generalizations to be used later. In other words, students benefit from discussion because it allows them to then apply the skills discussed to new problems. This aligns with what the participants noticed in their own classrooms.

Further, the participants described students benefiting from discourse with other students who could play the part of teacher for them. In other words, one student served as the MKO for the other student or students. When one student was positioned as the teacher in a given situation, they became the MKO. The participants did not describe any student as a "more knowledgeable other" specifically, but they perceived benefits to having one student serve as the teacher for others. This occurred in multiple ways: sometimes a student would lead the whole class, a student would help teach a small group, or a student would work with a partner and help a struggling student. Participants noted that these roles could change; students with different strengths would be the MKO in some situations and the learner in others. Participants believed that all students in the arrangement benefited because even the MKO would become more confident. They also believed that having a student as MKO rather than a teacher also helped make the classroom feel safer for taking risks and making mistakes.

Often, however, the MKO is described as an adult (Allman, 2022; Steele, 2001; Vygotsky, 1978), meaning there needs to be interactions between students and the teacher. Allman (2022) explains that children need to be guided through activities with the MKO, which then allows the children to be able to use strategies from those interactions. Many of the participants described using questioning as their opportunity to engage with students personally and to guide students. The participants would encourage students to explain their thought processes for solving problems and provide feedback to support them or introduce scaffolds if needed. Therefore, the participants used both students and adults as MKOs during discourse.

Overall, the participants described that students struggled to engage in mathematical discourse while using disciplinary language due to missing skills, but they also felt that gains were made throughout the school year because of their engagement in discourse. The participants perceived the need to prepare students for using disciplinary language by setting up scaffolds for most students. These scaffolds varied by student but often included sentence stems, word walls, and resource folders, among other strategies. The work done by the participants is supported by the sociocultural theory (Vygotsky, 1978), which emphasizes the use of constant communication, engagement with a MKO, and scaffolds within a student's ZPD.

In this study, the researcher interviewed ten viable participants about their experiences facilitating mathematical discourse and found that the participants perceived mathematical discourse as a pedagogical strategy positively, identified a range of research-based strategies to facilitate discourse, and supported students in using disciplinary language in ways supported by the sociocultural theory (Vygotsky, 1978). These perceptions provide an understanding of how elementary school teachers facilitate discourse and fill a gap in research regarding these perceptions. In identifying these perceptions, the research questions have been answered. In the next section, the implications of the research are explored.

Implications

The National Council of Teachers of Mathematics (NCTM) has sought mathematics education reform since 1980, culminating most recently in their text, *Principles to Actions* (2014). In this document, NCTM claimed that students needed to engage in discourse to learn mathematics. Just four years prior to the release of that document, the National Governors

Association Center for Best Practices and the Council of Chief State School Officers (2010) released the Common Core State Standards (CCSS), which included the use of mathematical discourse, as well. However, despite the demand for reform of mathematics education, national and international assessments continued to find students struggled in mathematics (NCES, 2022a; OECD, 2019) despite evidence that mathematical discourse improves mathematical achievement (Anderson, 2021; Bertolone-Smith & Gillette-Koyen, 2019; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021). It was thereby necessary to explore how teachers facilitate mathematical discourse, in part to understand what teachers' perceptions of mathematical discourse are, what strategies teachers use to facilitate it, and how teachers understand students' use of language while engaging in discourse.

Existing research often ignores teacher perceptions and focuses on the results of using mathematical discourse and suggests research-based strategies rather than on understanding how teachers facilitate it. The research does not examine what teachers believe they are doing. The purpose of this research study was to fill that gap and to explore the perceptions of public elementary K-5 educators regarding the use of mathematical discourse as a pedagogical strategy. The study examined the perceptions of teachers from schools in New England states that utilize the CCSS (Connecticut, Maine, New Hampshire, Rhode Island, and Vermont), as there is an expectation from the CCSS to facilitate discourse (National Governors Association Center for Best Practices and the Council of Chief State School Officers, 2010). Results indicated that teachers viewed mathematical discourse as a pedagogical strategy favorably; had a range of strategies to draw from, often from their curricular programs; and believed that a student's disciplinary language skills were a critical precursor to discourse. As a result, there are some implications for district or school officials stemming from the results of this study: (1) teachers need more professional development surrounding discourse; (2) teachers need curricular programs that provide them with strategies to facilitate discourse; and (3) teachers need more time for discourse.

Implication #1

The first implication formed from the results of this study is that teachers need more professional development surrounding discourse. The participants in the study all viewed mathematical discourse favorably and wanted to facilitate discourse. Each participant explained that they saw the benefits of discourse in their students' achievement and in their language development. However, the participants also described how this was a recent change of heart with the change of focus from product to process. Most of the participants described being hesitant to use more mathematical discourse as it was pushed upon them over the past couple of years, but they now viewed it as critical for student engagement, success, and even their own teaching. The participants who were especially hesitant about mathematical discourse were the ones who taught younger students, such as those in kindergarten, first, or second grade. Yet these participants still viewed mathematical discourse positively and described having success with their students. Much of this positivity was the result of the success with the strategies they had to facilitate discourse. However, many of these strategies came from professional development they sought outside of their own schools or districts.

Many of the participants in the study described reading books independently or in book studies, taking graduate-level classes, or signing up for professional workshops on mathematical discourse. While it is possible, but not explicitly stated, that the participants received continuing education credits for this work, it was suggested by the participants that their professional development was not provided by the districts. For example, the participants who took graduate classes did so on their own time. This implies that not every teacher in the school benefitted from the professional development, which could lead to disparities in the teachers' professional work with mathematical discourse. In fact, some of the participants explained that professional development was needed because of this disparity, stating that many of their coworkers still are hesitant about facilitating discourse. Two of the participants claimed that it needed to be stated how important it is that teachers be trained, as it helped change their own minds about mathematical discourse and contributed to their overall positive views.

Further, the strategies they had for facilitating discourse, many of which were research-based, often came from the professional development they had. In examining the sub-research question about strategies, many of the most successful strategies participants used, such as questioning, came from readings, graduate classes, or workshops participants had. This implies that much of the success teachers have facilitating discourse comes from professional development, which further implies that professional development is critical to the overall facilitation of discourse. If the participants wanted additional professional development, they would need to seek it out themselves, which provided little guidance or support from the district. Thus, professional development, as a critical basis for teacher facilitation of mathematical discourse, should be led by districts and schools to ensure consistency between educators and to ensure that teachers can be provided with additional guidance when needed.

Implication #2

The second implication formed from the results of the study is that teachers need curricular programs that provide them with strategies to facilitate discourse. Overall, the participants in the study felt supported by the curricular programs selected by their schools or districts. Each of the participants used, or were in the process of switching to, programs that had been reviewed by EdReports (2023b) as aligning to standards and meeting expectations of rigor and adherence to the mathematical practices. These programs included Illustrative, Bridges, Engage NY or Eureka Math, and Zearn. EdReports (2023b) lists additional programs that also meet these expectations. Because the participants had programs to use that supported mathematical discourse, they were almost guaranteed to facilitate it if they used the program with fidelity. Some of the programs, such as Bridges and Zearn, were described as needing additional supplemental strategies from teachers (EdReports, 2016; EdReports, 2021), which supports the need for professional development as explained above. However, overall, the programs' descriptions stated they valued collaboration and specifically created learning structures that encouraged students to communicate with others (Kendall Hall, 2021; The Math Learning Center, n.d.; Taylor, 2022; Zearn, 2021). Illustrative, which over half the participants used, even cited research from Vygotsky (1978) about the sociocultural theory in their design principles (Kendall Hunt, 2021).

The participants in the study described learning strategies for facilitating discourse from the programs they used in addition to their professional development. The learning structures built into the programs supported student engagement in discourse, while the teacher books provided suggestions for how the teachers could support the students in engaging in discourse. For many of the participants, it seemed they relied on their program to facilitate discourse, especially since many described using the programs with fidelity. For the participants who did not use their programs with fidelity, they still described taking many of their strategies from the program (i.e., "What do you notice, what do you wonder?" or "Which one doesn't belong?") even if they changed the tasks or overall structure of lessons with what they learned from their professional development. For that reason, it is critical that districts and schools select curricular programs that encourage discourse with research-based strategies.

However, it was evident that having a solid curricular program paired with professional development supported the educators in facilitating discourse the most. The participants felt they still needed professional development even when they had access to a high-quality program. One would fill in the gaps of the other. This further implies that the reason the educators viewed mathematical discourse so positively was because they felt successful in facilitating it because of the preparation they had to do so, namely the program and professional development they had.

Based on the experiences of the participants in this study, the professional development they sought on their own was perceived as more important in their ability to facilitate discourse, as they all described the need for professional development despite having strong programs. While this is the perception of participants, it is unclear whether professional development without a program is beneficial to teachers based on their experiences alone. There appears to be little research on the effect professional development has on teachers' abilities to facilitate discourse. A study by Chan et al. (2020) found that teachers who were provided professional development meant to improve the facilitation of mathematical discourse were more successful in doing so. It also resulted in increased mathematical achievement scores by students. A study by Mok et al. (2022) also found that student discourse increased with ninth and tenth grade students when their teachers were provided with professional development, but the increase was not statistically significant. Similarly, there is very little research on the effects of using a curricular program and the facilitation of mathematical discourse. Rather, the results of this study imply that both professional development and the curricular program are important in the teacher's ability to facilitate discourse based on the perceptions of the participants.

Implication #3

The final implication from the results of this study is that teachers need more time for discourse. Despite some of the limitations surrounding discourse, the participants in the study still viewed discourse as a worthwhile pedagogical strategy. The two limitations described by participants were time and professional development, the latter of which was addressed. The former limitation, time, provides another implication for district or school officials: teachers need more time. The participants described feeling they had to cut discussions short because they did not have enough time for math class. As a result, they felt they had missed opportunities for rich discussion. They also described feeling they had too many standards to teach without enough time to do so, which meant they could not adequately focus on teaching their students how to critique the reasoning of others, one of the mathematical practices (National Governors Association Center for Best Practices and the Council of Chief State School Officers, 2010) and an important component of mathematical discourse (Anderson, 2021; Bertolone-Smith & Gillette-Koyen, 2019; Ghousseini et al., 2021; Varhol et al., 2021).

Given that math is not the only subject that needs to be taught, ensuring there is enough time for math may be difficult to guarantee. However, the implication from this study is that there is not enough time set aside for mathematical instruction. It is therefore suggested that school master schedules be reviewed. It may be relevant to have classroom teachers be part of the discussion regarding instructional minutes. The participants described having between 50-80 minutes depending on the school and grade level. The longest amount of time any of the participants in this study had for math class was 80 minutes, though she said she rarely got to use the full 80 minutes because of transition periods surrounding the block. Bridges notes that it requires at least 80 minutes (The Math Learning Center, 2023), meaning that only one out of the

10 participants in this study potentially would have had enough time for a full Bridges lesson according to their master schedules.

Studies regarding instructional minutes do not agree how instructional time impacts mathematical proficiency or even how many instructional minutes should be provided for mathematics. A study by Temple and Mohammed (2020) examined the instructional time for seventh and eighth graders in Nebraska and found that seventh graders who received 90 minutes of instruction daily performed better than their peers who received 90 minutes every other day, yet there was no significant difference for eighth graders with the same parameters. A study by Heafner and Fitchett (2015) examined the instructional minutes of the core subjects in third grade and found that the average instructional minutes reported by teachers and administrators was approximately 70 minutes, making up about 18% of the weekly instructional time. Mullis et al. (2020) reported that, on average, schools allot approximately 17% of their instructional time for mathematics in countries that participate in the Trends in International Mathematics and Science Study (TIMSS). The United States ranked as the fifth highest in instructional hours devoted to mathematics instruction below Portugal, Italy, South Africa, and Singapore (Mullis et al., 2020). Portugal, Italy, and Singapore all performed better than the United States on the PISA, while South Africa did not participate (OECD, 2019). However, South Korea dedicated the fewest instructional hours to mathematics instruction yet still scored amongst the top countries in the world on the PISA (OECD, 2019). How important instructional time is for mathematics, and especially for mathematical discourse, is not fully understood. However, given that educators in the present study perceived not having enough time, it is worth reviewing how much time is allotted for mathematics instruction and to do a review of school schedules to ensure there is enough time to support mathematical discourse.

Overall, it appears that teachers want to facilitate discourse and view it as worthwhile. However, teachers need to be supported in this endeavor, especially since they view it as a recent change that differs greatly from past practices. Districts and schools need to consider what teachers need to help in the facilitation of mathematical discourse. Things like providing professional development opportunities, selecting programs that are aligned to standards and support the mathematical practices, and providing adequate time for math class are places to start. By beginning there, teachers believe they will be provided with a foundation to facilitate mathematical discourse successfully.

Recommendations for Action

The researcher is able to present some recommendations for action based on the purpose of the study, the existing research regarding the benefits of mathematical discourse, the data collected during the study, and the results of the study. These recommendations stem from the themes identified in the study, the connections to the research questions, and the implications resulting from the data. In general, teachers perceive mathematical discourse positively and want to facilitate it in their classrooms; however, they describe some factors that district administrators could address to better support their educators and students. One of the challenges participants in the study faced was finding adequate professional development offered at their schools or districts. The participants all found professional development outside of their districts through college courses, readings, or outside workshops. Districts need to offer staff professional development on mathematical discourse to ensure that staff receive similar messages regarding mathematical discourse and are being taught research-based strategies to facilitate discourse that are consistent across the school. Chen et al. (2020) found that teachers who received professional development on strategies to facilitate discourse had increased discourse in their classrooms and increased mathematical achievement from students. If professional development is provided by the district, every teacher would be able to access it and potentially apply it in their classrooms.

Further, districts also need to select programs for their schools that promote discourse. EdReports identifies programs that meet specific criteria through three gateways. According to EdReports (2023c), "Gateways 1 and 2 focus on questions of alignment" (para. 5), specifically to standards and depth of those standards, while Gateway 3 requires the programs to be user-friendly for both educators and students. There are currently 16 programs for mathematics K-5 that meet the expectations of these three gateways (Table 3), and 7 of those have perfect scores for the first two gateways (EdReports, 2023c).

Table 3

EdReports: Math Programs Meeting Expectations on Gateways 1 and 2

Program	Average Gateway 1 Score	Average Gateway 2 Score	Average Gateway 3 Score
Open Up Resources K-5 Math	14/14	18/18	25/27
Eureka Math[2]	14/14	18/18	24/27
Achievement First Mathematics	14/14	18/18	26.5/27
Imagine Learning Illustrative Mathematics K-5 Math	14/14	18/18	25/27
Kendall Hunt's Illustrative Mathematics	14/14	18/18	25/27
ORIGO Stepping Stones 2.0	14/14	16/18	24/27
Reveal Math	14/14	18/18	25/27
Zearn	14/14	16/18	33.5/38
enVision Mathematics Common Core	14/14	17/18	36/38
HMH Into Math	14/14	16.7/18	35/38
i-Ready Classroom Mathematics	14/14	17/18	38/38
Into Math Florida	14/14	16.5/18	35/38
enVision Florida Mathematics	14/14	17/18	36/38
Eureka Math	14/14	16/18	33/38
Ready	14/14	18/18	36/38
Bridges in Mathematics	12.5/14	16.3/18	37/38

Note. This includes programs meeting EdReport expectations as of the publication of this study.

Scores on EdReports are reported out for each individual grade level and can vary from grade to grade, so the average gateway scores presented in the table are the averages for only grades K-5. Districts should choose programs for their schools that meet the expectations of EdReports and that meet the needs of their students and educators. They should also ensure they read each full report while selecting a program. For example, while Bridges meets the expectations of EdReports, EdReports (2016) indicates that it does not fully support the depth of the mathematical practices and teachers may need to supplement to ensure students engage in discourse. If this is something the district feels it can support, Bridges may be an appropriate program. If district programs are not on the above list, it may be worth considering adopting a new program that is aligned with standards and supports mathematical discourse.

The researcher also recommends that districts and schools examine their master schedules to determine whether there is an appropriate amount of time dedicated to mathematics instruction. Because participants in the study thought they did not have enough time for mathematics instruction, they often felt they needed to reduce the amount of discourse in which students engaged or end conversations early. While research is mixed regarding the amount of instructional time needed for mathematics (Heafner & Fitchett, 2015; Mullis et al., 2020; Temple & Mohammed, 2020), it may be worth examining the instructional time allocations by subject on a case-by-case basis. For example, Bridges requires a minimum of 80 minutes of instructional time (The Math Learning Center, 2023), while lessons for Illustrative Math are designed for 50-minute class periods (Kendall Hunt, 2023). Schools should ensure their schedules meet these minimums; this may be a factor when selecting a program, as well.

While the districts and schools each play significant roles in helping educators facilitate discourse effectively, the classroom teachers are the ones engaging with students and have their own roles to play. Vygotsky (1978) explained in his sociocultural theory that learning is a social process. Educators need to ensure that they are engaging in discourse with their students daily. The participants in this study felt they had a greater concept of student understanding when they engaged in discourse with their students. Further, they felt that their students improved their mathematical achievement because of their engagement in discourse. One of the most important factors towards their success in discourse, however, was their classroom management. Classroom teachers who want to facilitate mathematical discourse need to ensure they establish clear expectations for discourse and ensure the classroom community is a safe learning place. Classroom teachers seeking to improve their ability to facilitate discourse need these two criteria in place.

Recommendations for Further Study

Through completing this study and analyzing the data, several gaps in existing research were identified, as well as the potential for further research through qualitative, quantitative, or mixed methods research. First, a qualitative study about the professional development opportunities offered by schools regarding mathematical discourse facilitation may be pertinent to determine what options exist for teachers. Further, studies examining the role of professional development and curricular programs on mathematical discourse are also needed. The use of curricular programs played a significant part in teachers' perceptions of mathematical discourse, so understanding what programs are used most frequently and how they are used should be further understood. A study examining the variable of time should also be conducted. For example, how does time allotment for mathematics alone impact the facilitation of mathematical discourse?

While this study explored the perceptions of educators in a phenomenological study, a mixed methods or quantitative study in which these perceptions are connected to the achievement of students may be pertinent to further address the problem of stagnant mathematics test scores (NCES, 2022a; OECD, 2019). The results of this study indicate that teachers perceived mathematical discourse as a pedagogical strategy positively, and they noted seeing perceived benefits related to mathematical achievement from their students. Participants described seeing increased engagement and overall improved mathematical outcomes, but there is no concrete evidence of this. It may be worthwhile to examine whether teachers who view mathematical discourse more favorably are associated with higher test scores via a quantitative study.

One theme that emerged from the study that deserves further research is Theme 2: classroom management impacts a teacher's ability to facilitate discourse. One of the participants described struggling greatly with facilitating mathematical discourse because of her challenges with student behaviors, despite her positive opinions surrounding mathematical discourse in general. Further studies may examine with greater detail the role classroom management practices have in facilitating mathematical discourse.

Several of the participants also mentioned some struggles in classroom management because of the COVID-19 pandemic. In addition to the academic troubles the participants perceived the students had, they also felt the students were behind socially, resulting in more difficult behaviors. These perceptions are supported by additional research (Robinson et al., 2023). Robinson et al. (2023) found that many teachers perceived academic achievement as a major concern following the pandemic. Furthermore, teachers may be dealing with more disruptive behaviors from students than before the pandemic. An examination of the connections between the COVID-19 pandemic and the language development of students, specifically in mathematics, could also provide greater information about how to facilitate mathematical discourse.

Additionally, Bloomberg and Volpe (2019) and Roberts and Hyatt (2019) describe limitations and delimitations as inherent to any study. The researcher identified several limitations and delimitations present in this study in Chapter 3. Limitations in this study include the researcher's own biases, which could have impacted the interpretations made in the results and the implications of the results. The small sample size of 10 viable participants can impact the findings of the results, as well. It is possible that the opinions of the 10 participants differ greatly from those of the masses. The backgrounds of these participants could also impact the findings. While the researcher sought participants from a range of grade levels and experience, the majority of participants taught fourth grade and only one participant had fewer than 10 years of experience. An additional, and perhaps the most significant, limitation relates to the sites used for recruitment. The researcher requested participation from teachers in Connecticut, Maine, New Hampshire, Rhode Island, and Vermont, all of which were represented in the study. With only five out of the 50 states represented, it is likely that the experiences and perceptions from the participants in these states do not match the experiences and perceptions of those in other states, especially given cultural and educational differences. Delimitations in the present study include the focus on the lived experiences of public elementary educators, and the use of a single semi-structured interview format.

Based on these limitations and delimitations, the researcher recommends that those interested in further study within this topic examine the topic on a larger scale such as utilizing a larger sample size. A selection of teachers more equally representing all grade levels would also be suggested for future studies. Most importantly, the researcher suggests that those interested in further study examine the experiences and perceptions of educators across the United States regarding mathematical discourse as a pedagogical strategy, even those that do not use the Common Core State Standards. This study examined the perceptions of teachers in states that expected teachers to facilitate discourse; it is worth examining the perceptions of teachers where this may not be the case.

By utilizing a single semi-structured interview format, the researcher ensured all participants were asked the same questions and could ask clarifying questions. Conducting follow-up interviews after transcribing the initial interviews may have allowed the researcher to gather more information that could have painted a fuller picture of the participants' perceptions. A researcher interested in further study may consider using follow up interviews with the participants to seek further clarification and gather additional information about experiences. This might provide a greater understanding of the perceptions of elementary teachers regarding the use of mathematical discourse as a pedagogical strategy.

Conclusion

The role of mathematical discourse over the past several decades has evolved thanks to the mathematics education reform started by the National Council of Teachers of Mathematics (NCTM). Mathematical discourse has become more of a necessity in mathematics education, with both the Common Core State Standards (CCSS; National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010) and NCTM (2014) requiring students to engage in it and teachers to facilitate it within the last 13 years. However, despite evidence suggesting that mathematical discourse leads to greater mathematical proficiency (Anderson, 2021; Bertolone-Smith & Gillette-Koyen, 2019; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021) and Vygotsky's (1978) sociocultural theory purporting that engagement in social interaction is how students learn, student achievement stagnated (NCES, 2022a; OECD, 2019).

There was also limited research regarding how teachers perceived they facilitated discourse, especially at the elementary level, suggesting a need for research in this area. Existing research supports mathematical discourse as beneficial for student mathematical proficiency levels (Anderson, 2021; Bertolone-Smith & Gillette-Koyen, 2019; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021). The literature also describes the best strategies for facilitating discourse, including questioning, critiquing of student work, establishing clear expectations, thoughtful positioning, scaffolding, and the selection of cognitively demanding tasks (Anderson, 2021; Bennett, 2014; Bertolone-Smith & Gillette-Koyen, 2019; Garcia et al., 2021; Ghousseini et al., 2021; Martin et al., 2015; Sigmon et al., 2022; Silva, 2021; Smith et al., 2020; Sullivan, 2019; Varhol et al., 2021; Wilson & Smith, 2022; Woods, 2021). Lastly, the literature also describes that mathematical discourse increases disciplinary language use (Croce & McCormick, 2019; Sigmon et al., 2022; Xu & Clarke, 2019).

This study sought to explore the perceptions of public elementary K-5 educators regarding the use of mathematical discourse as a pedagogical strategy, thus filling that gap in research. By interviewing, transcribing, and coding the interviews of ten viable participants, the study resulted in four themes: (1) teachers have a generally positive view of discourse as a pedagogical strategy; (2) classroom management impacts a teacher's ability to facilitate discourse; (3) teachers rely on curriculum and professional development for strategies to facilitate discourse; and (4) teachers view mathematical language as critical to student engagement in discourse. These themes addressed one main research question and two sub-questions:

Main Research Question: What are the perceptions of public elementary K-5 teachers regarding the use of mathematical discourse as a pedagogical strategy in the classroom?

Sub-Question 1. How do public elementary teachers describe how they use different pedagogical strategies for facilitating mathematical discourse?

Sub-Question 2. How do teachers describe how students engage in mathematical discourse while using disciplinary language?

The results of the study indicate that public elementary K-5 teachers view mathematical discourse as a pedagogical strategy favorably, with participants describing the strategy as a beneficial one for both them and students, even despite any limitations. Second, the strategies that the teachers use stem mostly from strategies that are research-based that they learned from professional development or that were embedded in their curricular programs. Third, teachers described the struggle many students faced utilizing disciplinary language and explained they used scaffolds as suggested by Vygotsky (1978) to support students. They believed that mathematical language was a crucial factor to student engagement in discourse, thus making those scaffolds for diverse learners even more valuable.

Recommendations for future research include an expanded study across the United States with a greater sample of participants and a study examining the relationship between the perceptions of teachers regarding mathematical discourse and the test scores of their students. This study fills a critical gap regarding the use of mathematical discourse and may help districts and schools support educators; this is especially important as a need for more professional development and time were suggested in the results of the study. This study also may provide teachers with additional strategies for facilitating discourse and encourage more teachers to adapt their practice to facilitate more mathematical discourse in their classrooms. In turn, more classrooms may see the facilitation of mathematical discourse as an everyday phenomenon.

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APPENDIX A: UNIVERSITY OF NEW ENGLAND INSTITUTIONAL REVIEW BOARD APPROVAL

UNE UNIVERSITY OF NEW ENGLAND
INNOVATION FOR A HEALTHIER PLANET

Office of Research Integrity
Institutional Review Board

Biddeford Campus
11 Hills Beach Road
Biddeford, ME 04005
(207) 602-2244 T
(207) 602-5905 F

Portland Campus
716 Stevens Avenue
Portland, ME 04103

DATE OF LETTER:	23 May 2023
PRINCIPAL INVESTIGATOR:	Christina Anderson
FACULTY ADVISOR:	Mitchell Henke, PhD
PROJECT NUMBER:	0523-15
RECORD NUMBER:	0523-15-01
PROJECT TITLE:	The Perceptions of Elementary Educators Regarding Mathematical Discourse Utilization in the Classroom: A Qualitative Phenomenological Study
SUBMISSION TYPE:	New Project
SUBMISSION DATE:	5/16/2023
ACTION:	Determination of Exempt Status
DECISION DATE:	5/23/2023
REVIEW CATEGORY:	Exemption Category # 2(ii)

The Office of Research Integrity has reviewed the materials submitted in connection with the above-referenced project and has determined that the proposed work is exempt from IRB review and oversight as defined by 45 CFR 46.104.

You are responsible for conducting this project in accordance with the approved study documents, and all applicable UNE policies and procedures.

If any changes to the design of the study are contemplated (e.g., revision to the research proposal summary, data collection instruments, interview/survey questions, recruitment materials, participant information sheet, and/or other approved study documents), the Principal Investigator must submit an amendment for review to ensure the requested change(s) will not alter the exempt status of the project.

If you have any questions, please send an e-mail to irb@une.edu and reference the project number as specified above within the correspondence.

Best Regards,

Bob Kennedy

Bob Kennedy, MS
Director of Research Integrity

APPENDIX B: EMAIL TO PARTICIPANTS

Dear [Teacher Name],

My name is Christina Anderson, and I am a doctoral candidate at the University of New England. As a doctoral candidate, I will be completing research on a topic of interest that aims to address an existing problem. I am choosing to complete my research on The Perceptions of Elementary Educators Regarding Mathematical Discourse Utilization in the Classroom: A Qualitative Phenomenological Study. For my data collection, I am seeking volunteers to participate in one interview of approximately 60 minutes via Zoom in June 2023.

By conducting this study, I hope to understand teacher perceptions of the use of mathematical discourse as a pedagogical strategy. The findings of my study may provide data about the use of mathematical discourse, or lack thereof, which may help schools understand mathematical achievement patterns.

Any participation is purely voluntary, and information gathered during the study cannot be traced back to the participant or individual school. Schools and teachers in the study will be provided with pseudonyms.

If you fit the following criteria, please consider reaching out to me.

1. You are over 18 years old.
2. You are a public-school teacher currently employed in an elementary school in grades K through 5.
3. You teach math as a general education teacher.
4. You have at least one year of experience at your current grade level.

If you fit the above criteria and are willing to volunteer to participate in my research, please send an email to canderson35@une.edu with your name, school, and grade level taught. I will follow up with additional information about the research and a participant information sheet.

Project Title: The Perceptions of Elementary Educators Regarding Mathematical Discourse Utilization in the Classroom: A Qualitative Phenomenological Study
Principal Investigator: Christina Anderson, canderson@sau39.org
Faculty Advisor: Dr. Mitch Henke, mhenke@une.edu

Thank you for your time, and I look forward to hearing from you!

Sincerely,
Christina Anderson
Doctoral Candidate, University of New England

APPENDIX C: PARTICIPANT INFORMATION SHEET

Participant Information Sheet

Version Date:	05/22/23
IRB Project #:	0523-15
Title of Project:	The Perceptions of Elementary Educators Regarding Mathematical Discourse Utilization in the Classroom: A Qualitative Phenomenological Study
Principal Investigator (PI):	Christina Anderson
PI Contact Information:	canderson35@une.edu, (603) 305-1916

INTRODUCTION

- This is a project being conducted for research purposes. Your participation is completely voluntary.
- The intent of the Participant Information Sheet is to provide you with important details about this research project.
- You are encouraged to ask any questions about this research project, now, during or after the project is complete.
- The use of the word 'we' in the Information Sheet refers to the Principal Investigator and/or other research staff.

WHAT IS THE PURPOSE OF THIS PROJECT?

The general purpose of this research project is to explore the perceptions of public elementary (K-5) educators regarding the use of mathematical discourse as a pedagogical strategy. 10 participants will be invited to participate in in this research as part of the Principal Investigator's (PI) dissertation research.

WHY ARE YOU BEING ASKED TO PARTICIPATE IN THIS PROJECT?

You are being asked to participate in this research project because you are age 18 or older; currently employed at a public elementary school; at least one year teaching mathematics at your current grade level to ensure some familiarity with the curriculum; and teach grades K-5.

WHAT IS INVOLVED IN THIS PROJECT?

You will be asked to participate in one semi-structured interview conducted by the Principal Investigator. The interview will take approximately 60 minutes over Zoom.

You will be given the opportunity to leave your camera on or off during the interview, and your interview will be recorded using Zoom.

You will be emailed a copy of your interview transcript to review for accuracy. You will have five business days to respond, or the PI will assume that you have no comments and assume the transcript to be accurate.

WHAT ARE THE POSSIBLE RISKS OR DISCOMFORTS INVOLVED FROM BEING IN THIS PROJECT?

The risks involved with participation in this research project are minimal and may include an invasion of privacy or breach of confidentiality. This risk will be minimized by using pseudonyms for each of the participants' names and by eliminating any identifying information from the study. Participants will have the opportunity to review their transcripts for accuracy and will be given the choice to have their cameras off during the interview. Participants have the right to skip or not answer any questions, for any reason.
Please see the 'WHAT ABOUT PRIVACY & CONFIDENTIALITY?' section below for additional steps we will take to minimize an invasion of privacy or breach of confidentiality from occurring.

WHAT ARE THE POSSIBLE BENEFITS FROM BEING IN THIS PROJECT?

There are no likely benefits to you by being in this research project; however, the information we collect may help us understand teacher perceptions of mathematical discourse as a pedagogical strategy and help explain the problem facing American students with their mathematical achievement today.

WILL YOU BE COMPENSATED FOR BEING IN THIS PROJECT?

You will not be compensated for being in this research project.

WHAT ABOUT PRIVACY AND CONFIDENTIALITY?

We will do our best to keep your personal information private and confidential. However, we cannot guarantee absolute confidentiality. Your personal information may be disclosed if required by law. Additionally, your information in this research project could be reviewed by representatives of the University such as the Office of Research Integrity and/or the Institutional Review Board.
The results of this research project may be shown at meetings or published in journals to inform other professionals. If any papers or talks are given about this research, your name will not be used. We may use data from this research project that has been permanently stripped of personal identifiers in future research without obtaining your consent.
The following additional measures will be taken to protect your privacy and confidentiality:

- Data will only be collected during one-on-one participant interviews using Zoom. No information will be taken without the participant's consent. Transcribed interviews will be checked by participants for accuracy before they are added to the study.
- Pseudonyms will be used for all participants and their schools, and any personally identifying information will be stripped from the interview transcript.
- All names, affiliated schools, grade levels, emails, and years of experience gathered during the recruitment will be recorded and linked to a uniquely assigned pseudonym within a master list. The participants' associated schools will also be assigned a pseudonym within a master list.

- The master list will be kept securely and separately from the study data and will be accessible only to the Principal Investigator.
- The interview will be conducted in a private setting to ensure others cannot hear the conversation.
- Participants are given the option to turn off their camera during the Zoom interview.
- Once member checking of the transcribed interview is complete, the recorded Zoom interview will be destroyed. Once all transcripts have been verified by the participants, the master list of personal information will be destroyed.
- All other study data will be retained on record for 3 years after the completion of the project and then destroyed. The study data may be accessed upon request by representatives of the University (e.g., faculty advisors, Office of Research Integrity, etc.) when necessary.
- All data collected will be stored on a password-protected USB accessible only by the Principal Investigator in a locked file cabinet.

WHAT IF YOU WANT TO WITHDRAW FROM THIS PROJECT?

You have the right to choose not to participate, or to withdraw your participation at any time until the Master List is destroyed without penalty or loss of benefits. You will not be treated differently if you decide to stop taking part in this project.

If you request to withdraw from this project, the data collected about you would be destroyed when the Master List is in existence; the researcher may not be able to do so after the Master List is destroyed.

WHAT IF YOU HAVE QUESTIONS ABOUT THIS PROJECT?

You have the right to ask, and have answered, any questions you may have about this research project. If you have questions about this project, complaints, or concerns, you should contact the Principal Investigator listed on the first page of this document.

WHAT IF YOU HAVE QUESTIONS ABOUT YOUR RIGHTS AS A RESEARCH PARTICIPANT?

If you have questions or concerns about your rights as a research participant, or if you would like to obtain information or offer input, you may contact the Office of Research Integrity at (207) 602-2244 or via e-mail at irb@une.edu.

APPENDIX D: INTERVIEW PROTOCOL AND QUESTIONS

The following table provides the interview protocol for the study.

Interview Protocol

Script prior to interview:
Thank you so much for joining me and being willing to participate in this interview. I mentioned to you before that my study is about teacher perceptions of mathematical discourse. To elaborate, the purpose of my study is to explore the perceptions of public elementary K-5 educators regarding use of mathematical discourse as a pedagogical strategy, where mathematical discourse is defined by the National Council of Teachers of Mathematics (NCTM; 2010) as "the mathematical communication that occurs in a classroom" (p. 1). In essence, I am exploring how teachers are using mathematical discourse and how they perceive they use it as a teaching tool. Our interview today will last approximately 30-60 minutes, during which I will ask you about your experiences using mathematical discourse in your instruction, what strategies you use to promote mathematical discourse, and how you encourage the use of mathematical language.

I mentioned prior that I would be recording this interview for both audio and video. Do I still have your permission to do so?
_____ yes   _____ no

<u>If yes:</u> *Thank you. Please let me know at any point if you would like me to stop recording or if you would like to keep something you said off the record. You may also choose to have your camera off or on at any point.*

<u>If no:</u> *Thank you for letting me know. I will only take notes of our discussion.*

Before we begin, do you have any questions? [Discuss questions.]
If you have any questions during the interview or study, please ask at any time!

Main Research Question: What are the perceptions of public elementary teachers regarding the use of mathematical discourse as a pedagogical strategy in the classroom?

	Personal experiences in the classroom with discourse	**Mathematical discourse knowledge**
1. I would like to begin this interview by asking you what you believe mathematical discourse to be. How would you describe mathematical discourse?		X
2. How do you feel about the use of discussion in a math class?	X	X
3. Describe your experiences encouraging mathematical	X	

discourse in the classroom. If I was observing a typical class session, what would I see and hear?		
4. Do you notice while teaching that students are communicating mathematically? If yes: *How do you use this to drive your instruction?* If no: *Why do you think that is?*	X	
5. How does using discourse in the classroom affect your students' performance in your classroom?	X	X
Sub-Research Question #1: How do public elementary teachers describe how they use different pedagogical strategies for facilitating mathematical discourse?		
	Describing their own pedagogical strategies	**Evidence-based strategies**
6. So now, I would like to get an idea of the techniques you use to support mathematical discourse in your class. What do you do to encourage students to communicate with each other during your math class?	X	
7. What do you do to encourage students to communicate with you?	X	
8. Do you set up expectations for your class discussions ahead of time? If yes: *How do you come up with these expectations?* If no: *How do you monitor discussions?*	X	X
9. How might you use questioning strategies to guide a conversation during class?	X	X
10. How do you pick what questions you are going to ask?	X	

11. While students engage in mathematical discourse, what questions might you ask to guide them towards what you want them to discover or notice about a topic?	X	
12. Mathematical Practice 3 of the Common Core states that students need to be able to construct viable arguments and critique the reasoning of others. How do you use discourse to encourage this behavior?	X	X
13. If you noticed there was one particularly shy student who wasn't engaging in the conversation, how would you react to engage them?	X	X
14. Suppose a student or students were really struggling with a concept. How do you provide support to them?	X	X
15. How do you incorporate choice?	X	X
16. How do you select tasks that will serve as the focus of your discussions?	X	X
Sub-Research Question #2: How do teachers describe how students engage in mathematical discourse while using disciplinary language?		
	Mathematical context	**Types of responses**
17. How do you encourage students to talk like mathematicians during class discussions?	X	
18. For students who are not ready to use the language verbally, how do you encourage them to participate in other ways?		X
19. For students who struggle with difficult, real-world tasks, how do you continue to encourage them to use and interpret the language	X	

without removing the context of the problem?		
Before we conclude the interview, is there anything about mathematical discourse that you feel you did not have a chance to say or that you would like to add?		